

A mixed Algorithm for Stochastic Multiple Objective Programming Problems Using MATLAB Program

Tarek H. M. Abou-El-Enien

Department of Operations Research & Decision Support, Faculty of Computers & Artificial Intelligence,
Cairo University, Postal Code 12613, Giza, Egypt

t.hanafy@fci-cu.edu.eg

Abstract:

The purpose of this work is to obtain nondominated solutions for stochastic multiple objective programming (SMOP) problems using the chance constrained programming technique and the weighting approach. A mixed algorithm for solving the SMOP problems was proposed. The proposed algorithm was tested. A MATLAB[®] program is constructed to solve the SMOP problems. The hybrid algorithm can be used to solve the different SMOP problems and it can also be applied by scientists and engineers.

Keywords: MATLAB[®], Multiple objective programming problems, stochastic parameters, weighting method, chance constrained programming method.

1. Introduction

Stochastic Multiple objective programming (SMOP) problems, [5], are an important class in optimization, [1,2,7]. The target of this work is to introduce a computational approach using MATLAB[®] (R2018a), [1,2,6], to solve linear SMOP problems based on the chance constrained programming technique, [5,6], and the weighting approach, [5].

In the following section, formulation of Linear SMOP problem containing stochastic numbers in the RHS of the constraints and basic definition are presented. A mixed algorithm is given in section (3). A traditional solution of numerical example is introduced in section (4). A MATLAB[®] program and its execution for the given example is introduced in section (5). Section (6) presents conclusions.

2. LMOP Problem with Stochastic Numbers in the RHS of the Constraints

Suppose the following LMOP problem containing stochastic numbers in the RHS of the constraints:

$$\text{Maximize } Q(Y) = (q_1(X), q_2(Y), \dots, q_k(Y)) \quad (1-1)$$

subject to

$$X \in M = \{X \in \mathbb{R}^n : P\{\sum_{t=1}^n b_{lj}x_j \leq v_l\} \geq \alpha_l, \quad l = 1, 2, \dots, m, \\ y_t \geq 0, \quad t = 1, 2, \dots, n\}. \quad (1-2)$$

where

k : no. of objective functions,

m : no. of constraints,

n: no. of variables,

b_{lj} : coefficients of the LHS of the constraints, $l = 1, 2, \dots, m, t = 1, 2, \dots, n$

\mathbb{R} : the set of all real numbers,

Y: an n-dimensional column vector of variables,

$\mathbb{N} = \{1, 2, \dots, n\}$,

$$\mathbb{R}^n = \{X = (x_1, x_2, \dots, x_n)^T : x_i \in \mathbb{R}, i \in \mathbb{N}\}.$$

In addition, P means probability, $\alpha_l, l = 1, 2, \dots, m$, is a specified probability level. Suppose that the random parameter, $v_i, i = 1, 2, \dots, m$, is distributed normally and independently of each other with known mean $E\{v_i\}$ and variance $Var\{v_i\}$.

Using the chance constrained programming method [5,7], problem (1) have the deterministic form:

$$\text{Maximize } (q_1(Y), q_2(Y), \dots, q_k(Y)) \tag{2-1}$$

subject to

$$Y \in M' = \{Y \in \mathbb{R}^n: \sum_{t=1}^n b_{it}y_t \leq E\{v_i\} + k_{\alpha_i}\sqrt{Var\{v_i\}}, l = 1, 2, \dots, m, y_t \geq 0, t = 1, 2, \dots, n\} \tag{2-2}$$

where $k_{\alpha_l}, l = 1, \dots, m$, is the standard normal value such that $\Phi(k_{\alpha_l}) = 1 - \alpha_l, l = 1, \dots, m$ and Φ represents the cumulative distribution function of the standard normal distribution. The nondominated solution to problem (2) can be defined as follows, [2]:

Definition:

A point $Y^* \in \mathbb{X}$ is said to be a nondominated solution to problem (2), iff \nexists another $Y \in \mathbb{X}$, such that $q_r(Y) \geq q_r(Y^*), (r = 1, \dots, k)$, with strictly inequality holding for at least one r .

3. Mixed Algorithm for solving linear SMOP problem

To obtain a nondominated solution to problem (1), a mixed algorithm using the chance constrained programming technique [6,7] and the weighting approach, [5], is presented as follows:

Mixed Algorithm:

(1): Using chance constrained programming method, [5,7], to transform problem (1) to the form of problem (2):

(2): Use the weighting approach, [7,8], to transform problem (2) to the following problem :

$$\text{Maximize } \sum_{r=1}^k w_r q_r(Y) \tag{3-1}$$

subject to

$$Y \in M' = \{Y \in \mathbb{R}^n: \sum_{t=1}^n b_{it}y_t \leq E\{v_i\} + k_{\alpha_i}\sqrt{Var\{v_i\}}, l = 1, \dots, m, y_t \geq 0, t = 1, \dots, n\} \tag{3-2}$$

where $w_r \geq 0, r = 1, \dots, k, \sum_{r=1}^k w_r = 1$.

(3): Suppose $E\{v_i\} = E^*\{v_i\}, Var\{v_i\} = Var^*\{v_i\}, i = 1, 2, \dots, m$.

(4): Use the statistical tables (Table of normal distribution function), [7], or the calculator to find $k_{\alpha_i}, i=1, 2, \dots, m$.

(5): Let $w_r = w_r^* \geq 0$, then use the simplex method to solve problem (3).

(6): Let Y^* is the optimal to problem (3):

- (i) If $w_r > 0, \forall r$, thus Y^* is a nondominated solution, go to (7)
- (ii) If $w_r \geq 0, \forall r$, and X^* is a unique for problem (3), thus Y^* is an efficient solution, go to (7)
- (iii) If $w_r \geq 0, \forall r$, and \exists another solutions, use the nondominated test, [5].

(7): Stop.

4. Numerical Example

Consider the linear SMOP problem containing stochastic numbers in the RHS of the constraints, (let $w_1^* = w_2^* = 0.5$) :



$$\text{Maximize } [(3x_1 + 5x_2), (-2x_1 + 3x_2)]$$

subject to

(4)

$$P\{x_1 + x_2 \leq b_0\} \geq 0.7257,$$

$$P\{x_1 \leq b_1\} \geq 0.5,$$

$$P\{2x_2 \leq b_2\} \geq 0.4013,$$

$$x_1, x_2 \geq 0,$$

where

$$E(b_0) = 8, E(b_1) = 2, E(b_2) = 7, \text{Var}(b_0) = 25, \text{Var}(b_1) = 4 \text{ and } \text{Var}(b_2) = 16.$$

Apply the mixed algorithm to solve the given ex.

Solution.:

Use the table of normal distribution function, [7], to find $k_{\alpha_i}^*$, $i=1,2,\dots,m$.

$$\text{Thus, } k_{\alpha_1}^* = -0.06, k_{\alpha_2}^* = 0, k_{\alpha_3}^* = 0.25$$

Use problem (2) to write the following problem:

$$\text{Maximize } [(3x_1 + 5x_2), (-2x_1 + 3x_2)]$$

subject to

$$x_1 + x_2 - 8 - (-0.6)(\sqrt{25}) \leq 0,$$

$$x_1 - 2 - (0)(\sqrt{4}) \leq 0,$$

$$2x_2 - 7 - (0.25)(\sqrt{16}) \leq 0,$$

$$x_1, x_2 \geq 0.$$

which can be written as follows:

$$\text{Maximize } [(3x_1 + 5x_2), (-2x_1 + 3x_2)]$$

subject to

$$x_1 + x_2 \leq 5, \quad x_1 \leq 2, \quad 2x_2 \leq 8, \quad x_1, x_2 \geq 0.$$

Apply the weighting approach as follows:

$$\text{Maximize } Z = w_1z_1 + w_2z_2 = [0.5(3x_1 + 5x_2) + 0.5((-2x_1 + 3x_2))]$$

subject to

$$x_1 + x_2 \leq 5, \quad x_1 \leq 2, \quad x_2 \leq 4, \quad x_1, x_2 \geq 0.$$

By solving the above LP problem, the optimal is : $Z^* \cong 16.5$, $x_1^* \cong 1$, $x_2^* \cong 4$.

Also, to find a nondominated solution to the above ex., a MATLAB program based on the mixed algorithm is given in the following section.

5. MATLAB Program for Linear SMOP problem:

In this section, a MATLAB[®] program and its execution for the given ex. is introduced.

w1=0.5;

w2=0.5;

obj1=[3, 5];

obj2=[2, -3];

LHS_C=[1 1; 1 0; 0 2];

```

RHS_C=[0;0;0];
%E_b is the expected value of each stochastic number in a constraint
%var_b is the variance of each stochastic number in a constraint
%p is the value that gets the value of e from the normal distribution table
E_b0=8; var_b0=25; p0=0.7257;
E_b1=2; var_b1=4; p1=0.5;
E_b2=7; var_b2=16; p2=0.4013;
E0 = -norminv(p0);
E1 = -norminv(p1);
E2 = -norminv(p2);
fprintf('Absolute value of normal distribution of p of 1st stochastic number is:')
E0
fprintf('Absolute value of normal distribution of p of 2nd stochastic number is:')
E1
fprintf('The absolute value of the normal distribution of p of the 3rd stochastic number is:')
E2
obj=-1*w1*obj1 + w2*obj2;
%calculate the new RHS of the constraints
RHS_C=[E_b0+(E0*sqrt(var_b0));E_b1+(E1*sqrt(var_b1));E_b2+(E2*sqrt(var_b2))];
RHS_C
lb=zeros(2,1);
[x,fval]=linprog(obj,LHS_C,RHS_C,[],[],lb);
fprintf('The value of optimal obj. fun.')
z=-fval
fprintf('The values of optimal decision variables')
x

```

RUN

The absolute value of the normal distribution of p of the 1st stochastic number is:

E0 =
-0.5999

Absolute value of normal distribution of p of 2nd stochastic number is:

E1 =
0

Absolute value of normal distribution of p of 3rd stochastic number is:

E2 =
0.2500

RHS_C =

5.0007

2.0000

7.9999

'The value of optimal obj. fun.:

z =

16.5002

'The values of optimal decision variables

x =

1.0007

4.0000

The following table presents a comparison between the hand solutions of the given ex. by the mixed algorithm and the solutions by the MATLAB program.

Table (1): Comparison between the hand solutions and the MATLAB solutions.

	Hand solutions	MATLAB solutions
Z^*	16.5	16.5002
x_1^*	1	1.0007
x_2^*	4	4.0000

6. Conclusions

In this work, a mixed algorithm and a MATLAB program using chance constrained programming technique and the weighting approach to obtain nondominated solutions for stochastic Multiple objective programming (SMOP) problems is introduced. By comparing the results: the hand solutions of the given ex. by the mixed algorithm and the solutions by the program is approximately identical.

The engineers and scientists can apply the given MATLAB program and the mixed algorithm to solve various practical SMOP problems.

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Author Biography



Prof. Dr. Tarek H. M. Abou-El-Enien is a Professor of Mathematical Programming, - Department of O.R. & D. S. - Faculty of Computers & Artificial Intelligence - Cairo University. His research interests in decision analysis, soft computing, modeling, data mining and multi-level multi-objective Programming field.