An Introduction To Complex Arithmetic Calculus And An Original Reformulation Of The Goldbach Conjecture.

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Abstract

Prime numbers are well known (for simple characterizations of primes via divisibility, see [11] and [12] and [13] and [14] and [15]), and the Goldbach conjecture (see [1] or [2] or [3] or [4] or [5] or [6] or [7] or [8] or [9] or [10] or [16]) states that every even integer $e \geq 4$ is of the form $e = p + q$, where q and p are prime. In this paper, we give an original reformulation of the Goldbach conjecture via complex arithmetic calculus. This reformulation shows that the Goldbach conjecture can be attacked without using strong investigations that have been on this conjecture in the past

AMS Classification 2000: $05xx$ **and** $11xx$**.**

Keywords: goldbach, goldbachian.

Introduction

Prologue. This paper is divided into three sections. In Section.**1**, we introduce definitions that are not standard and we present some elementary properties deduced from these definitions. In Section.**2**, using definitions of Section.**1**, we give the trivial reformulation of the Goldbach conjecture that we will use in Section.**3**. In Section.**3**, we prove a proposition linked to complex arithmetic calculus and we use it to give an original reformulation of the Goldbach conjecture via the trivial reformulation of Section.**2**. This original reformulation shows that the Goldbach conjecture can be attacked without using strong investigations that have been on this conjecture in the past, and by using only complex arithmetic calculus.

1. Non-standard definitions and simple properties.

Definition 1.0. We say that e is *goldbach*, if e is an even integer ≥ 4 and is of the form $e = p + q$, where p and q are prime. Note that the Goldbach conjecture (see Abstract) states that every even integer $e \geq 4$ is *goldbach*.
Example 1.0.0. 4 is *goldbach*, since 4 is an even integer ≥ 4 and $4 = 2 + 2$, where 2 is prime; 6 is *goldbach*, since 6 is an even integer ≥ 4 and $6 = 3 + 3$, where 3 is prime; 8 is *goldbach*, since 8 is an even integer ≥ 4 and $8 = 3 + 5$, where 3 and 5 are prime; 10 is *goldbach*, since 10 is an even integer ≥ 4 and $10 = 3 + 7$, where 3 and 7 are prime; 12 is *goldbach*, since 12 is an even integer ≥ 4 and $12 = 7 + 5$, where 5 and 7 are prime; and 1764 is also *goldbach*, because 1764 is an even integer ≥ 4 and is of the form $1764 = 883 + 881$, where 883 and 881 are prime.

That being so, let us define:

Definition 1.1. We say that e is *goldbachian*, if e is an even integer ≥ 4 and if every even integer v such that $4 \leq v \leq e$ is of the form $v = p_v + q_v$, where p_v and q_v are prime; *in other worlds*, we say that e is *goldbachian*, if e is an even integer ≥ 4 and if every even integer v with $4 \leq v \leq e$ is *goldbach* (see Definition 1.0 for the meaning of *goldbach*); *in other terms again*, we say that e is *goldbachian*, if e is an even integer ≥ 4 and v is an even integer of the form $4 \leq v \leq e$, *implies that* v is *goldbach*. Using the previous definition, then we have the following trivial remarks.

Remark 1.1.0 12 *is golbachian.*
Proof. Indeed, observe (by using Example 1.0.0 of Definition 1.0) that 12 is an even integer ≥ 4, and every even integer v of the form $4 \leq v \leq 12$ is *goldbach*; consequently 12 is *goldbachian.*

Remark 1.1.1. *If d is goldbachian and if d' is an even integer of the form $4 \leq d' \leq d$, then d' is also goldbachian.*
Proof. Immediate and is a trivial consequence of the definition of *goldbachian* introduced above.

Remark 1.1.2. *12 and 10 and 8 and 6 and 4 are simultaneously goldbachian.*
Proof. Immediate and is a trivial consequence of Remark 1.1.0 and Remark 1.1.1.

Remark 1.1.3. *For every integer $n \in \{1, 2, 3, 4, 5\}$, $2n + 2$ is goldbachian .*
Proof. Immediate and is a trivial consequence of Remark 1.1.2.

Note that *goldbachian* implies *goldbach*; so there is no confusion between *goldbachian* and *goldbach.* Having defined *goldbach* and *goldbachian*, then it comes:

Definitions 1.2. For every integer $n \geq 2$, we define $\mathcal{G}(n)$, g_n, $\mathcal{G}(n+1)$ and g_{n+1} as follows:

$\mathcal{G}(n) = \{g; 1 < g \leq 2n, \textit{ and } g \textit{ is goldbachian}\}$, and $g_n = \max_{g \in \mathcal{G}(n)} g$. Using the definitions of $\mathcal{G}(n)$ and g_n, then it becomes trivial to deduce that for every integer $n \geq 1$, we clearly have
$\mathcal{G}(n+1) = \{g; 1 < g \leq 2n+2, \textit{ and } g \textit{ is goldbachian}\}$, and $g_{n+1} = \max_{g \in \mathcal{G}(n+1)} g$. It is immediate that $\mathcal{G}(n) \subseteq \mathcal{G}(n+1)$ for every integer $n \geq 2$, and therefore $g_n \leq g_{n+1}$ for every integer $n \geq 2$. Using the previous definitions, then we have the following trivial remarks.

Remark 1.2.0 *If $n \geq 2$, then $\mathcal{G}(n) \subseteq \mathcal{G}(n+1)$ and $g_n \leq g_{n+1}$.* **Proof**. Immediate (it suffices to use the definitions of $\mathcal{G}(n)$, $\mathcal{G}(n+1)$, g_n and g_{n+1}).

Remark 1.2.1. *If $g_{n+1} \neq 2n+2$, then $\mathcal{G}(n+1) = \mathcal{G}(n)$ and $g_{n+1} = g_n$.* **Proof**. Immediate and is a trivial consequence of the definition of $(\mathcal{G}(n), g_n, \mathcal{G}(n+1), g_{n+1})$ introduced above.

Remark 1.2.2. *If $g_{n+1} \leq 2n$, then $\mathcal{G}(n+1) = \mathcal{G}(n)$ and $g_{n+1} = g_n$.* **Proof**. Observe that $g_{n+1} \neq 2n+2$ and use Remark 1.2.1.

Remark 1.2.3. *For every integer $n \in \{1, 2, 3, 4, 5\}$, we have $g_{n+1} = 2n + 2$.* **Proof**. Immediate and is a trivial consequence of Remark 1.1.3 and the definition of g_{n+1}.

Now using the definitions of $\mathcal{G}(n+1)$ and g_{n+1}, then the following Proposition becomes trivial.

Proposition 1.3. *Let n be an integer ≥ 2. We have the following seven trivial properties.*

(1.3.0.) g_{n+1} *is even and* $g_{n+1} \leq 2n+2$.
(1.3.1.) $g_{n+1} = 2n+2$, *if and only if,* $2n+2$ *is goldbachian* $\Big($ *in other words,* $g_{n+1} \neq 2n+2$, *if and only if,* $2n+2$ *is not goldbachian* $\Big)$.
(1.3.2.) $g_n \leq g_{n+1}$.
(1.3.3.) If $g_{n+1} < 2n+2$, *then* $2n+2$ *is not goldbachian.*
(1.3.4.) If $2n+2 \leq e$ *and if* e *is goldbachian, then* $2n+2$ *is goldbachian.*
(1.3.5.) (An implicite using of the Goldbach formula). If $g_{n+1} < 2n+2$, then $2n+2$ is not goldbachian and there exists an integer e such that $1 \leq e \leq n$ and $2e+2$ can not be of the form $2e+2 = p_e + q_e$, where p_e and q_e are prime.
(1.3.6.) (An explicite using of the Goldbach formula). $g_{n+1} = 2n+2$, if and only if, for every integer n' such that $1 \leq n' \leq n$, we have $2n'+2 = p_{n'} + q_{n'}$, where $p_{n'}$ and $q_{n'}$ are prime.

Proof. Properties (1.3.0) and (1.3.1) are immediate (it suffices to use the definition of g_{n+1}). Property (1.3.2) is trivial (it suffices to use the definition of g_{n+1} via the definition of g_n); and property (1.3.3) is a trivial consequence of property (1.3.1). Property (1.3.4) is an immediate consequence of Remark 1.1.1 of Definition 1.1. Property (1.3.5) is trivial $\Big($ it suffices to use property (1.3.3) and the definition of goldbachian (see Definition 1.1) $\Big)$, and property (1.3.6) is an immediate consequence of the definition of goldbachian and the definition of g_{n+1} (see Definition 1.1 for goldbachian and Definitions 1.2 for g_{n+1}).

We will use g_{n+1} to give the trivial reformulation of the Goldbach conjecture.

2. The trivial reformulation of the Goldbach conjecture.

Theorem 2.1. *The following are equivalent.*

(1). For every integer $n' \geq 1$,we have $2n' + 2 = p_{n'} + q_{n'}$, where $p_{n'}$ and $q_{n'}$ are prime.
(2) The Goldbach conjecture is true [i.e. every even integer $e \geq 4$ is of the form $e = p_e + q_e$, where p_e and q_e are prime].
(3) For every integer $n \geq 1$, $2n + 2$ is goldbachian.
(4) For every integer $n \geq 1$, we have $g_{n+1} = 2n + 2$.

Proof. (1) $\Rightarrow$ (2)] Immediate [since property (2) is only the obvious reformulation of property (1)] **;** (2) $\Rightarrow$ (3)] Immediate [it suffices to use the meaning of the Goldbach conjecture and the definition of goldbachian]**;**
(3) $\Rightarrow$ (4)] Immediate [it suffices to use the definition of goldbachian and the definition of g_{n+1}]**;** (4) $\Rightarrow$ (1)] Immediate, by using property (1.3.6) of Proposition 1.3.

Theorem 2.1 is the trivial reformulation of the Goldbach conjecture. Theorem 2.1 will help us in Section.**3** to give an original reformation of the Goldbach conjecture via complex arithmetic calculus. Before, we need the following elementary combinatoric remark.

Remark 2.2.*Let n be an integer ≥ 1; consider $\mathcal{G}(n + 1)$ and g_{n+1} (see Definitions 1.2). We have the following four properties.*

(2.2.0.) g_{n+1} is even and $4 \leq g_{n+1} \leq 2n + 2$.
(2.2.1.) **If** *$g_{n+1} \neq 2n + 2$, then: $n > 5$ and $g_{n+1} = g_n$.*
(2.2.2.) (An implicite using of the Goldbach formula). If $g_{n+1} \neq 2n + 2$, then: $n > 5$ and $g_{n+1} = g_n$ and there exists an integer e such that $1 \leq e \leq n$ and $2e + 2$ can not be of the form $2e + 2 = p + q$, where p and q are prime.
(2.2.3.) (Another implicite using of the Goldbach formula). If $g_{n+1} \leq 2n$, then: $n > 5$ and $g_{n+1} = g_n$ and there exists an integer e such that $1 \leq e \leq n$ and $2e + 2$ can not be of the form $2e + 2 = p + q$, where p and q are prime.

Proof. Property (2.2.0) is immediate. Indeed, it is immediate (by using the definition of g_{n+1}) that g_{n+1} is even. It is trivial that 4 is goldbachian (use Remark 1.1.2 of Definition 1.1) and $4 = 2(1 + 1)$; so $4 \in \mathcal{G}(n + 1)$ and therefore $g_{n+1} \geq 4$. It is immediate that $g_{n+1} \leq 2n + 2$ (use the definition of g_{n+1}). Now using the previous two inequalities, then it becomes trivial to deduce that $4 \leq g_{n+1} \leq 2n + 2$. Property (2.2.0) folows. Property (2.2.1) is also immediate. Indeed, **if** $g_{n+1} \neq 2n + 2$, clearly $n > 5$ (since $g_{n+1} = 2n + 2$ for $n \in \{1, 2, 3, 4, 5\}$, by using Remark 1.2.3 of Definitions 1.2), and clearly

$$\mathcal{G}(n + 1) = \mathcal{G}(n) \text{ and } g_{n+1} = g_n$$

(observe that $g_{n+1} \neq 2n + 2$ and use Remark 1.2.2 of Definitions 1.2). Property (2.2.1) follows. Property (2.2.2) is only the trivial reformulation of property (2.2.1), by using the definition of g_{n+1} and g_n (see Definitions 1.2). Propoperty (2.2.3) is an immediate consequence of propoperty (2.2.2) $\Big($ Indeed observe that $g_{n+1} \neq 2n + 2$ and use propoperty (2.2.2) $\Big)$. Remark 2.2 immediately follows.
We will use Remark 2.2 in Section.**3** to give an original reformulation of the Goldbach conjecture via complex arithmetic calculus.

3. Properties linked to complex arithmetic calculus and an original reformation of the Goldbach conjecture.

In this section, we prove a proposition linked to complex arithmetic calculus and we use it to give an original reformulation of the Goldbach conjecture. This original reformulation via complex arithmetic calculus shows that the Goldbach conjecture can be attacked without using strong investigations that have been done on this conjecture in the past. Before, we need the following last definition.

Definition 3.0 **(Fundamental)**. Let n be an integer ≥ 1 and let g_{n+1} (see Definitions 1.2); then ϕ_n is defined as follows.

$$\phi_n = (\, i g_{n+1}^3 - 2ing_{n+1}^2 + 2n\,)^2, \;\; i^2 = -1.$$

It is immediate that for every integer $n \geq 1$, ϕ_n is well defined and gets sense. Now using Definition 3.0, then we have the following elementary Proposition linked to complex arithmetic calculus.

Proposition 3.1 *Let n be an integer ≥ 2 and let g_{n+1} (see Definitions 1.2); now look at ϕ_n introduced in Definition 3.0, and via ϕ_n, consider ϕ_{n-1} (this consideration gets sense, since $n \geq 2$, and therefore $n - 1 \geq 1$).* **If** *$g_{n+1} \neq 2n + 2$, then we have the following two simple properties.*
(3.1.0.) (Implicite using of the Goldbach formula). $g_{n+1} = g_n$ and there exists an integer e such that $1 \leq e \leq n$ and

$2e+2$ can not be of the form $2e+2 = p+q$, where p and q are prime.
(3.1.1.)

$$\phi_{n-1} - \phi_n = -4g_{n+1}^5 - 4g_{n+1}^4 - 4ig_{n+1}^3 - 8ig_{n+1}^2 - 8n + 4 + 8ng_{n+1}^4 + 16ing_{n+1}^2.$$

Proof. (3.1.0). Indeed observing (by the hypotheses) that $g_{n+1} \neq 2n+2$, clearly $g_{n+1} = g_n$ and there exists an integer e such that $1 \leq e \leq n$ and $2e+2$ can not be of the form $2e+2 = p+q$, where p and q are prime. (use property (2.2.2) of Remark 2.2). Property (3.1.0) follows.
(3.1.1). Indeed, look at ϕ_n, and observe (by Definition 3.0) that

$$\phi_n = (\, ig_{n+1}^3 - 2ing_{n+1}^2 + 2n\,)^2 \qquad (3.1).$$

Now let $n-1$ and look at ϕ_{n-1}; then using equality *(3.1)*, it becomes trivial to deduce that

$$\phi_{n-1} = (\, ig_n^3 - 2i(n-1)g_n^2 + 2(n-1)\,)^2 \qquad (3.2).$$

Noticing (by property (3.1.0)) that $g_{n+1} = g_n$ and using the preceding equality, then it becomes trivial to deduce that equality *(3.2)* clearly says that

$$\phi_{n-1} = (\, ig_{n+1}^3 - 2i(n-1)g_{n+1}^2 + 2(n-1)\,)^2 \qquad (3.3).$$

It is elementary to see that equality *(3.3)* clearly says that

$$\phi_{n-1} = (\, ig_{n+1}^3 - 2ing_{n+1}^2 + 2n + 2ig_{n+1}^2 - 2)^2 \qquad (3.4).$$

Look at equality *(3.4)*; observing (by elementary computation) that

$$(\, ig_{n+1}^3 - 2ing_{n+1}^2 + 2n + 2ig_{n+1}^2 - 2)^2 = \lambda_n \qquad (3.5),$$

where

$$\lambda_n = (\, ig_{n+1}^3 - 2ing_{n+1}^2 + 2n\,)^2 + 2(2ig_{n+1}^2 - 2)(\, ig_{n+1}^3 - 2ing_{n+1}^2 + 2n\,) + (2ig_{n+1}^2 - 2)^2 \qquad (3.5'),$$

then, using equalities *(3.5)* and *(3.5′)* it becomes trivial to deduce that equality *(3.4)* says that

$$\phi_{n-1} = (\, ig_{n+1}^3 - 2ing_{n+1}^2 + 2n\,)^2 + 2(2ig_{n+1}^2 - 2)(\, ig_{n+1}^3 - 2ing_{n+1}^2 + 2n\,) + (2ig_{n+1}^2 - 2)^2 \qquad (3.6).$$

Using equality *(3.1)*, then it becomes elementary do deduce that equality *(3.6)* says that

$$\phi_{n-1} = \phi_n + 2(2ig_{n+1}^2 - 2)(\, ig_{n+1}^3 - 2ing_{n+1}^2 + 2n\,) + (2ig_{n+1}^2 - 2)^2 \qquad (3.7).$$

Observing (by elementary computation and the fact that $i^2 = -1$) that

$$2(2ig_{n+1}^2 - 2)(\, ig_{n+1}^3 - 2ing_{n+1}^2 + 2n\,) + (2ig_{n+1}^2 - 2)^2 = \lambda'_n \qquad (3.8),$$

where

$$\lambda'_n = -4g_{n+1}^5 - 4g_{n+1}^4 - 4ig_{n+1}^3 - 8ig_{n+1}^2 - 8n + 4 + 8ng_{n+1}^4 + 16ing_{n+1}^2 \qquad (3.8'),$$

then, using equalities *(3.8)* and *(3.8′)*, it becomes trivial to deduce that equality *(3.7)* says that

$$\phi_{n-1} = \phi_n - 4g_{n+1}^5 - 4g_{n+1}^4 - 4ig_{n+1}^3 - 8ig_{n+1}^2 - 8n + 4 + 8ng_{n+1}^4 + 16ing_{n+1}^2 \qquad (3.9).$$

Using equality *(3.9)*, then we immediately deduce that

$$\phi_{n-1} - \phi_n = -4g_{n+1}^5 - 4g_{n+1}^4 - 4ig_{n+1}^3 - 8ig_{n+1}^2 - 8n + 4 + 8ng_{n+1}^4 + 16ing_{n+1}^2.$$

Property (3.1.1) follows and Proposition 3.1 immediately follows.

Having proved the previous simple Proposition linked to complex arithmetic calculus, we are now ready to give an original reformulation of the Goldbach conjecture.

Theorem 3.2 (An original reformulation of the Goldbach conjecture). *The following are equivalent.*

(1) The Goldbach conjecture is true [i.e. every even integer $e \geq 4$ is of the form $e = p+q$, where p and q are prime].
(2) For every integer $n \geq 2$

$$\phi_{n-1} - \phi_n \neq -4g_{n+1}^5 - 4g_{n+1}^4 - 4ig_{n+1}^3 - 8ig_{n+1}^2 - 8n + 4 + 8ng_{n+1}^4 + 16ing_{n+1}^2.$$

Proof. (1) ⇒ (2)]. Observe (by remarking that the Goldbach conjecture is true and by using Theorem 2.1) that

$$For\ every\ integer\ n \geq 1, we\ have\ g_{n+1} = 2n+2 \qquad (3.10).$$

Using the definition of g_{n+1}, then it becomes trivial to deduce that *(3.10)* clearly implies that

$$For\ every\ integer\ n \geq 2, we\ have\ g_n = 2n \qquad (3.11).$$

Now look at ϕ_n introduced in Definition 3.0; then using equality of *(3.10)*, it becomes trivial to deduce that

$$\phi_n = (\, 2ig^2_{n+1} + 2n\,)^2 \qquad (3.12).$$

That being so, consider ϕ_{n-1} (this consideration gets sense, since $n \geq 2$, and therefore $n-1 \geq 1$), then using equality of *(3.11)*, it becomes trivial to deduce that

$$\phi_{n-1} = (\, ig^3_n - 2i(n-1)g^2_n + 2(n-1)\,)^2 = (\, 2n + 2ig^2_n - 2\,)^2 \qquad (3.13).$$

Using equalities *(3.12)* and *(3.13)*, then it becomes trivial to check (by elementary computation and the fact that $i^2 = -1$ and by using *(3.10)* and *(3.11)*) that

$$\phi_{n-1} - \phi_n \neq -4g^5_{n+1} - 4g^4_{n+1} - 4ig^3_{n+1} - 8ig^2_{n+1} - 8n + 4 + 8ng^4_{n+1} + 16ing^2_{n+1}.$$

(1) ⇒ (2)] Otherwise (we reason by reduction to absurd), let n be an integer ≥ 1 such that

$$g_{n+1} \neq 2n + 2 \qquad (3.14)$$

(observe that such a n exists, by remarking that the Goldbach conjecture is false and by using Theorem 2.1). Clearly

$$n > 5 \qquad (3.15)$$

(use *(3.14)* and property (2.2.1) of Remark 2.2). Using *(3.14)* and *(3.15)*, then it becomes immediate to deduce all the hypotheses of Proposition 1.3 are satisfied for such a n; therefore, all the conclusion of Proposition 1.3 are satisfied for such a n; in particular, property (3.1.1) of Proposition 1.3 is satisfied for such a n. Consequently

$$\phi_{n-1} - \phi_n = -4g^5_{n+1} - 4g^4_{n+1} - 4ig^3_{n+1} - 8ig^2_{n+1} - 8n + 4 + 8ng^4_{n+1} + 16ing^2_{n+1},$$

and the previous equality gives rise to a serious contradiction.
Theorem 3.2 is an original reformulation of the Goldbach conjecture via complex arithmetic calculus and is stronger than all the investigations that have been done on the Goldbach conjecture in the past. Indeed, Theorem 3.2 clearly says that: **if** for every integer $n \geq 2$, $\phi_{n-1} - \phi_n \neq -4g^5_{n+1} - 4g^4_{n+1} - 4ig^3_{n+1} - 8ig^2_{n+1} - 8n + 4 + 8ng^4_{n+1} + 16ing^2_{n+1}$, then the Goldbach conjecture immediately follows. Visibly, Theorm 3.2 is not related to all the investigations that have been done on the Goldbach conjecture in the past and can be trasformed to attack the Goldbach conjecture in an original way.

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