Contra $n\mathcal{I}_{g\mu}$-continuity

Selvaraj Ganesan

Assistant Professor, PG & Research Department of Mathematics,
Raja Doraisingam Government Arts College, Sivagangai-630561, Tamil Nadu, India.
(Affiliated to Alagappa University, Karaikudi, Tamil Nadu, India)

sgsgsgsgsg77@gmail.com

Abstract

In this paper, $n\mathcal{I}_{g\mu}$-closed sets and $n\mathcal{I}_{g\mu}$-open sets are used to define and investigate a new class of maps called $n\mathcal{I}_{g\mu}$-continuous, $n\mathcal{I}_{g\mu}$-irresolute map and contra $n\mathcal{I}_{g\mu}$-continuous maps in nano ideal topological spaces. We discuss the relationship with some other related maps.

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1 Introduction

Let (O, $\mathcal{N}$, $\mathcal{I}$) be an nano ideal topological space with an ideal $\mathcal{I}$ on O, where $\mathcal{N} = \tau_R(X)$ and $(.)^*_n : \wp(\text{O}) \to \wp(\text{O})$ ($\wp$(O) is the set of all subsets of O) [10, 11]. For a subset A $\subseteq$U, A^*_n ($\mathcal{I}$, $\mathcal{N}$) = {x $\in$ O : $\text{G}_n \cap$ A $\notin \mathcal{I}$, for every $\text{G}_n \in \text{G}_n$(x)}, where G_n = {G_n | x $\in \text{G}_n$, $\text{G}_n \in \mathcal{N}$} is called the nano local function (brielfy n-local function) of A with repect to $\mathcal{I}$ and $\mathcal{N}$. We will simply write A^*_n for A^*_n ($\mathcal{I}$, $\mathcal{N}$). Parimala et al [11] introduced the concept of nano ideal generalized closed sets in nano ideal topological spaces and investigated some of its basic properties. Recently, Ganesan [5] introduced and studied contra $n\mathcal{I}_g$-continuity in nano ideal topological spaces. In this paper, we introduce the notation of $n\mathcal{I}_{g\mu}$-continuous, $n\mathcal{I}_{g\mu}$-irresolute map and contra $n\mathcal{I}_{g\mu}$-continuity in nano ideal topological spaces and discuss their properties and give various characterizations.

2 Preliminaries

2.1 Definition

[10, 11] A subset A of a nano ideal topological space (O, $\mathcal{N}$, $\mathcal{I}$) is said to be n$\star$-closed if $\text{A}^*_n \subseteq$ A.

2.2 Lemma

[10, 11] Let (O, $\mathcal{N}$, $\mathcal{I}$) be an nano topological space with an ideal $\mathcal{I}$ and A$\subseteq\text{A}^*_n$, then A^*_n=n-cl(A^*_n)=n-cl(A)

2.3 Definition

[9] A subset M of a space (U, $\tau_R(X)$) is said to be nano pre-closed set if ncl(nint(M)) $\subseteq$ M. nano pre-open if its complemnt is nano pre-closed.

2.4 Definition

[4] A subset M of a space (U, $\tau_R(X)$) is said to be a ngμ-closed set if nclo(A) $\subseteq$ U whenever A $\subseteq$ U and U is nμ-open in (U, $\tau_R(X)$). The complement of ngμ-closed set is called ngμ-open set.

2.5 Definition

A subset A of an nano ideal topological space (U, $\mathcal{N}$, $\mathcal{I}$) is said to be

1. nano-$\mathcal{I}$-generalized closed (briefly, $n\mathcal{I}_g$-closed) [11] if A$^*_n$ $\subseteq$ V whenever A$\subseteq$ V and V is n-open. $n\mathcal{I}_g$-open if its complement is $n\mathcal{I}_g$-closed
2. $n\mathcal{I}_{g\mu}$-closed [2] if A$^*_n$ $\subseteq$ V whenever A$\subseteq$ V and V is ngμ-open. $n\mathcal{I}_{g\mu}$-open if its complement is $n\mathcal{I}_{g\mu}$-closed.

2.6 Definition

[8] An nano topological space (U, $\tau_R(X)$) is said to nano locally indiscrete if every n-open set is n-closed.

2.7 Definition

[1] A nano topological space (U, $\tau_R(X)$) is said to be nano-regular space, if for each nano closed set F and each point x $\notin$ F , there exists disjoint nano open sets G and H such that x $\in$ G and F $\subset$ H

2.8 Definition

[7] A nano topological space (U, $\tau_R(X)$) is said to be nano-connected if (U, $\tau_R(X)$) cannot be expressed as a disjoint union of two non-empty nano-open sets. A subset of (U, $\tau_R(X)$) is nano-connected as a subspace. A subset is said to be nano disconnected if and only if it is not nano-connected

2.9 Definition

[4] A function f: (O, $\mathcal{N}$) $\rightarrow$ (P, $\mathcal{N}'$) is said to be contra ngμ-continuous if f^{-1}(V) is a ngμ-closed set of (O, $\mathcal{N}$) for every n-open set V of (P, $\mathcal{N}'$).

2.10 Definition

A map f: (K, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (L, $\mathcal{N}'$, $\mathcal{J}$) is said to be n$\star$-continuous [6] if f^{-1}(A) is n$\star$-closed in (K, $\mathcal{N}$, $\mathcal{I}$) for every n-closed set A of (L, $\mathcal{N}'$, $\mathcal{J}$).

2.11 Theorem

1. Every n-closed is n$\star$-closed set but not conversely [3].
2. Every n$\star$-closed set is $n\mathcal{I}_{g\mu}$-closed but not conversely [2]
3. Every ng-closed set is $n\mathcal{I}_{g\mu}$-closed but not conversely [2]

2.12 Definition

[5] A map f: (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (P, $\mathcal{N}'$, $\mathcal{J}$) is said to be contra n$\star$-continuous (resp. contra $n\mathcal{I}_g$-continuous) if f^{-1}(G) is a n$\star$-closed (resp. $n\mathcal{I}_g$-closed) in (O, $\mathcal{N}$, $\mathcal{I}$) for every n-open set G of (P, $\mathcal{N}'$, $\mathcal{J}$).

3 $n\mathcal{I}_{g\mu}$-Continuous Map

3.1 Definition

[2] A map f: (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (P, $\mathcal{N}'$, $\mathcal{J}$) is called $n\mathcal{I}_{g\mu}$-continuous if f^{-1}(V) is a $n\mathcal{I}_{g\mu}$-closed set of (O, $\mathcal{N}$, $\mathcal{I}$) for every n-closed set V of (P, $\mathcal{N}'$, $\mathcal{J}$).

3.2 Proposition

Every n$\star$-continuous is $n\mathcal{I}_{g\mu}$-continuous but not conversely.
Proof The proof follows from Theorem 2.11 (2).□

3.3 Example

Let O = {8, 9, 10}, with O/ R= {{10}, {8, 9}, {9, 8}} and X= {8, 9}. Then the Nano topology $\mathcal{N}$ = {ϕ, {8, 9}, O} and $\mathcal{I}$={$\emptyset$, {8}}. Let P = {8, 9, 10}, with P/ R= {{8}, {9, 10}} and X = {8, 9}. Then the Nano topology $\mathcal{N}'$ = {ϕ, {8}, {9, 10}, P} and $\mathcal{J}$={$\emptyset$, {8}}. Then n$\star$-closed sets are ϕ, O, {8}, {10}, {8, 10} and $n\mathcal{I}_{g\mu}$-closed sets are ϕ, O, {8}, {10}, {8, 10}, {9, 10}. Define f: (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (P, $\mathcal{N}'$, $\mathcal{J}$) be the identity map. Then f is $n\mathcal{I}_{g\mu}$-continuous but not n$\star$-continuous, since f^{-1}({9, 10})= {9, 10} is not n$\star$-closed in (O, $\mathcal{N}$, $\mathcal{I}$).

3.4 Proposition

Every $n\mathcal{I}_{g\mu}$-continuous is $n\mathcal{I}_g$-continuous but not conversely.
Proof The proof follows from Theorem 2.11 (3). □

3.5 Example

Let O = {8, 9, 10}, with O/ R= {{8}, {9, 10}} and X= {8}. Then the Nano topology $\mathcal{N}$ = {ϕ, {8}, O} and $\mathcal{I}$={∅, {8}}. Let P = {8, 9, 10}, with P/ R= {{9}, {8, 10}, {10, 8}} and X = {8, 10}. Then the Nano topology $\mathcal{N}'$ = {ϕ, {8, 10}, P} and $\mathcal{J}$={∅}. Then $n\mathcal{I}_{g\mu}$-closed sets are ϕ, O, {8}, {9, 10} and $n\mathcal{I}_{g}$-closed sets are ϕ, O, {8}, {9}, {10}, {8, 9}, {8, 10}, {9, 10}. Define f: (O, $\mathcal{N}$, $\mathcal{I}$) → (P, $\mathcal{N}'$, $\mathcal{J}$) be the identity map. Then f is $n\mathcal{I}_{g}$-continuous but not $n\mathcal{I}_{g\mu}$-continuous, since f^{-1}({9})= {9} is not $n\mathcal{I}_{g\mu}$-closed in (O, $\mathcal{N}$, $\mathcal{I}$).

3.6 Remark

The composition of two $n\mathcal{I}_{g\mu}$-continuous maps need not be $n\mathcal{I}_{g\mu}$-continuous and this is shown from the following example.

3.7 Example

Let O, $\mathcal{N}$, $\mathcal{I}$ and f as in the Example 3.5. Let P = {8, 9, 10}, with P/ R= {{8}, {9, 10}} and X= {8, 10}. Then the Nano topology $\mathcal{N}'$ = {ϕ, {8}, {9, 10}, P} and $\mathcal{J}$={∅}. Then $n\mathcal{I}_{g\mu}$-closed sets are ϕ, P, {8}, {9}, {10}, {8, 9}, {8, 10}, {9, 10}. Let Q = {8, 9, 10} with Q/ R= {{10}, {8, 9}, {9, 8}} and X= {8, 9}. Then the Nano topology $\mathcal{N}'_*$= {ϕ, {8, 9}, Q} and $\mathcal{K}$ = {∅}. Define g : (P, $\mathcal{N}'$, $\mathcal{J}$) → (Q, $\mathcal{N}'_*$, $\mathcal{K}$) be the identity maps. Clearly f and g are contra $n\mathcal{I}_{g\mu}$-continuous but their g ∘ f : (O, $\mathcal{N}$, $\mathcal{I}$) → (Q, $\mathcal{N}'_*$, $\mathcal{K}$) is not contra $n\mathcal{I}_{g\mu}$-continuous, because V = {1, 2} is n-closed in (Q, $\mathcal{N}'_*$) but (g ∘ f $^{-1}$({10})= f^{-1}(g^{-1}({10})) = f^{-1}({10}) = {10}, which is not $n\mathcal{I}_{g\mu}$-closed in (O, $\mathcal{N}$, $\mathcal{I}$).

3.8 Proposition

Let f: (O, $\mathcal{N}$, $\mathcal{I}$) → (P, $\mathcal{N}'$, $\mathcal{J}$) is $n\mathcal{I}_{g\mu}$-continuous if and only if f^{-1}(U) is $n\mathcal{I}_{g\mu}$-open in (O, $\mathcal{N}$, $\mathcal{I}$) for every n-open set U in (P, $\mathcal{N}'$, $\mathcal{J}$).

Proof Let f: (O, $\mathcal{N}$, $\mathcal{I}$) → (P, $\mathcal{N}'$, $\mathcal{J}$) be $n\mathcal{I}_{g\mu}$-continuous and U be an n-open set in (P, $\mathcal{N}'$, $\mathcal{J}$). Then U^c is n-closed in (P, $\mathcal{N}'$, $\mathcal{J}$) and since f is $n\mathcal{I}_{g\mu}$-continuous, $f^{-1}(U^c)$ is $n\mathcal{I}_{g\mu}$-closed in (O, $\mathcal{N}$, $\mathcal{I}$) . But $f^{-1}(U^c) = f^{-1}((U))^c$ and so f^{-1}(U) is $n\mathcal{I}_{g\mu}$-open in (O, $\mathcal{N}$, $\mathcal{I}$) .

Conversely, assume that f^{-1}(U) is $n\mathcal{I}_{g\mu}$-open in (O, $\mathcal{N}$, $\mathcal{I}$) for each n-open set U in (P, $\mathcal{N}'$, $\mathcal{J}$). Let F be a n-closed set in (P, $\mathcal{N}'$, $\mathcal{J}$). Then F^c is n-open in (P, $\mathcal{N}'$, $\mathcal{J}$) and by assumption, $f^{-1}(F^c)$ is $n\mathcal{I}_{g\mu}$-open in (O, $\mathcal{N}$, $\mathcal{I}$) . Since $f^{-1}(F^c) = f^{-1}((F))^c$, we have f^{-1}(F) is n-closed in (O, $\mathcal{N}$, $\mathcal{I}$) and so f is $n\mathcal{I}_{g\mu}$-continuous. □ We introduce the following definition

3.9 Definition

A map f: (O, $\mathcal{N}$, $\mathcal{I}$) → (P, $\mathcal{N}'$, $\mathcal{J}$) is called $n\mathcal{I}_{g\mu}$-irresolute if f^{-1}(V) is a $n\mathcal{I}_{g\mu}$-closed set of (O, $\mathcal{N}$, $\mathcal{I}$) for every $n\mathcal{I}_{g\mu}$-closed set V of (Y, σ, $\mathcal{J}$).

3.10 Theorem

Every $n\mathcal{I}_{g\mu}$-irresolute map is $n\mathcal{I}_{g\mu}$-continuous but not conversely.

Proof Let f: (O, $\mathcal{N}$, $\mathcal{I}$) $\to$ (P, $\mathcal{N}'$, $\mathcal{J}$) be a $n\mathcal{I}_{g\mu}$-irresolute map. Let V be a n-closed set of (P, $\mathcal{N}'$, $\mathcal{J}$). Then by the Theorem 2.11 (1) and (2), V is $n\mathcal{I}_{g\mu}$-closed. Since f is $n\mathcal{I}_{g\mu}$-irresolute, then f^{-1}(V) is a $n\mathcal{I}_{g\mu}$-closed set of (O, $\mathcal{N}$, $\mathcal{I}$) . Therefore f is $n\mathcal{I}_{g\mu}$-continuous. □

3.11 Example

Let O, $\mathcal{N}$, $\mathcal{I}$ and f as in the Example 3.5. Let P = {8, 9, 10}, with P/ R= {{8}, {9, 10}, {10, 9}} and X = {9, 10}. Then the Nano topology $\mathcal{N}'$ = {ϕ, {9, 10}, P} and $\mathcal{J}$={∅, {9}}. Then $n\mathcal{I}_{g\mu}$-closed sets are ϕ, P, {8}, {9}, {8, 9}, {8, 10}. (i) Because V = {8} is n-closed on (P, $\mathcal{N}'$, $\mathcal{J}$) it is clear that f^{-1}({8}) ={8} is $n\mathcal{I}_{g\mu}$-closed set of (O, $\mathcal{N}$, $\mathcal{I}$) . (ii) It is clear that {8, 9} is $n\mathcal{I}_{g\mu}$-closed set of (P, $\mathcal{N}'$, $\mathcal{J}$) but f^{-1}({8, 9}) ={8, 9} is not a $n\mathcal{I}_{g\mu}$-closed set of (O, $\mathcal{N}$, $\mathcal{I}$) . Thus f is not $n\mathcal{I}_{g\mu}$-irresolute map. However f is $n\mathcal{I}_{g\mu}$-continuous map.

3.12 Theorem

Let f: (O, $\mathcal{N}$, $\mathcal{I}$) $\to$ (P, $\mathcal{N}'$, $\mathcal{J}$) and g: (P, $\mathcal{N}'$, $\mathcal{J}$) $\to$ (Q, $\mathcal{N}'_*$, $\mathcal{K}$). be any two maps. Then

1. g ∘ f is $n\mathcal{I}_{g\mu}$-continuous if g is n⋆-continuous and f is $n\mathcal{I}_{g\mu}$-continuous.
2. g ∘ f is $n\mathcal{I}_{g\mu}$-irresolute if both f and g are $n\mathcal{I}_{g\mu}$-irresolute.
3. g ∘ f is $n\mathcal{I}_{g\mu}$-continuous if g is $n\mathcal{I}_{g\mu}$-continuous and f is $n\mathcal{I}_{g\mu}$-irresolute.

Proof (1) Since g is a n⋆-continuous from (P, $\mathcal{N}'$, $\mathcal{J}$) $\to$ (Q, $\mathcal{N}'_*$, $\mathcal{K}$), for any n-closed set q as a subset of Q, we get g^{-1}(q) = G is a n-closed set in (P, $\mathcal{N}'$, $\mathcal{J}$). As f is a $n\mathcal{I}_{g\mu}$-continuous map. We get $(g \circ f)^{-1}$(q)= $f^{-1}(g^{-1}(q))$ = f^{-1}(G) = S and S is a $n\mathcal{I}_{g\mu}$-closed set in (O, $\mathcal{N}$, $\mathcal{I}$) . Hence (g ∘ f) is a $n\mathcal{I}_{g\mu}$-continuous map.
(2) Consider two $n\mathcal{I}_{g\mu}$-irresolute maps, f: (O, $\mathcal{N}$, $\mathcal{I}$) $\to$ (P, $\mathcal{N}'$, $\mathcal{J}$) and g : (P, $\mathcal{N}'$, $\mathcal{J}$) $\to$ (Q, $\mathcal{N}'_*$, $\mathcal{K}$) is a $n\mathcal{I}_{g\mu}$-irresolute maps. As g is consider to be a $n\mathcal{I}_{g\mu}$-irresolute map, by Definition 3.9, for every $n\mathcal{I}_{g\mu}$-closed set q ⊆ (Q, $\mathcal{N}'_*$, $\mathcal{K}$), g^{-1}(q) = G is a $\mathcal{I}_{\ddot{g}}$-closed in (P, $\mathcal{N}'$, $\mathcal{J}$). Again since f is $n\mathcal{I}_{g\mu}$-irresolute, $(g \circ f)^{-1}$(q)= $f^{-1}(g^{-1}(q))$ = f^{-1}(G) = S and S is a $n\mathcal{I}_{g\mu}$-closed set in (O, $\mathcal{N}$, $\mathcal{I}$) . Hence (g ∘ f) is a $n\mathcal{I}_{g\mu}$-irresolute map.
(3) Let g be a $n\mathcal{I}_{g\mu}$-continuous map from (P, $\mathcal{N}'$, $\mathcal{J}$) $\to$ (Q, $\mathcal{N}'_*$, $\mathcal{K}$) and q subset of Q be a n-closed set. Therefore g^{-1}(q) is a $n\mathcal{I}_{g\mu}$-closed set in (P, $\mathcal{N}'$, $\mathcal{J}$), by Theorem 2.11 (1) and (2), g^{-1}(z) = G is a $n\mathcal{I}_{g\mu}$-closed set in (P, $\mathcal{N}'$, $\mathcal{J}$). Also since f is $n\mathcal{I}_{g\mu}$-irresolute, we get $(g \circ f)^{-1}$(q)= $f^{-1}(g^{-1}(q))$ = f^{-1}(G) = S and S is a $n\mathcal{I}_{g\mu}$-closed set in (O, $\mathcal{N}$, $\mathcal{I}$) . Hence (g ∘ f) is a n$n\mathcal{I}_{g\mu}$-continuous map. □

4 Contra $n\mathcal{I}_{g\mu}$-continuity

We introduce the following definition

4.1 Definition

A map f: (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (P, $\mathcal{N}'$, $\mathcal{J}$) is said to be contra $n\mathcal{I}_{g\mu}$-continuous if f^{-1}(G) is a $n\mathcal{I}_{g\mu}$-closed set of (O, $\mathcal{N}$, $\mathcal{I}$) for every n-open set G of (P, $\mathcal{N}'$, $\mathcal{J}$).

4.2 Proposition

Every contra n$\star$-continuous map is contra $n\mathcal{I}_{g\mu}$-continuous.

Proof Let f: (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (P, $\mathcal{N}'$, $\mathcal{J}$) be a contra n$\star$-continuous map and let G be any n-open set in (P, $\mathcal{N}'$, $\mathcal{J}$). Then, f^{-1}(G) is n$\star$-closed in O. Since every n$\star$-closed set is $n\mathcal{I}_{g\mu}$-closed, f^{-1}(G) is $n\mathcal{I}_{g\mu}$-closed in O. Therefore f is contra $n\mathcal{I}_{g\mu}$-continuous. □

4.3 Example

Let O, $\mathcal{N}$, $\mathcal{I}$, P, $\mathcal{N}'$, $\mathcal{J}$ and f as in the Example 3.3. Then f is contra $n\mathcal{I}_{g\mu}$-continuous but not contra n$\star$-continuous, since f^{-1}({9, 10})= {9, 10} is not n$\star$-closed in (O, $\mathcal{N}$, $\mathcal{I}$).

4.4 Proposition

Every contra $n\mathcal{I}_{g\mu}$-continuous map is contra $n\mathcal{I}_{g}$-continuous.

Proof Let f: (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (P, $\mathcal{N}'$, $\mathcal{J}$) be a contra $n\mathcal{I}_{g\mu}$-continuous map and let G be any n-open set in (P, $\mathcal{N}'$, $\mathcal{J}$). Then, f^{-1}(G) is $n\mathcal{I}_{g\mu}$-closed in O. Since every $n\mathcal{I}_{g\mu}$-closed set is $n\mathcal{I}_{g}$-closed, f^{-1}(G) is $n\mathcal{I}_{g}$-closed in O. Therefore f is contra $n\mathcal{I}_{g}$-continuous. □

4.5 Example

Let O, $\mathcal{N}$, $\mathcal{I}$, P, $\mathcal{N}'$, $\mathcal{J}$ and f as in the Example 3.5. Then f is contra $n\mathcal{I}_{g}$-continuous but not contra $n\mathcal{I}_{g\mu}$-continuous, since f^{-1}({8, 10})= {8, 10} is not $n\mathcal{I}_{g\mu}$-closed in (O, $\mathcal{N}$, $\mathcal{I}$).

4.6 Proposition

Every contra ngμ-continuous map is contra $n\mathcal{I}_{g\mu}$-continuous.

Proof Let f: (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (P, $\mathcal{N}'$, $\mathcal{J}$) be a contra ngμ-continuous map and let G be any n-open set in (P, $\mathcal{N}'$, $\mathcal{J}$). Then, f^{-1}(G) is ngμ-closed in O. Since every ngμ-closed set is $n\mathcal{I}_{g\mu}$-closed, f^{-1}(G) is $n\mathcal{I}_{g\mu}$-closed in O. Therefore f is contra $n\mathcal{I}_{g\mu}$-continuous. □

4.7 Example

Let O , $\mathcal{N}$, $\mathcal{I}$ and f be defined as an Example 3.5. Let P = {8, 9, 10}, with P/ R= {{8}, {9, 10}} and X= {8, 10}. Then the Nano topology $\mathcal{N}'$ = {ϕ, {8}, {9, 10}, P} and $\mathcal{J}$={$\emptyset$, {9}}. Then ngμ-closed sets are ϕ, O, {9, 10}. Then f is contra $n\mathcal{I}_{g}$-continuous but not contra ngμ-continuous, since f^{-1}({8})= {8} is not ngμ-closed in (O, $\mathcal{N}$, $\mathcal{I}$).

4.8 Remark

The following example shows that ngμ-continuity and contra $n\mathcal{I}_{g\mu}$-continuity are independent.

4.9 Example

Let O , $\mathcal{N}$, $\mathcal{I}$ and f be defined as an Example 3.3. Let P = {8, 9, 10}, with P/ R= {{8}, {9, 10}, {10, 9}} and X= {9, 10}. Then the Nano topology $\mathcal{N}'$ = {ϕ, {9, 10}, P} and $\mathcal{J}$={$\emptyset$}. Then ngμ-closed sets are ϕ, O, {10}, {8, 10}, {9, 10}. Then f is contra $n\mathcal{I}_{g\mu}$-continuous but not ngμ-continuous, since f^{-1}({8})= {8} is not ngμ-closed in (O, $\mathcal{N}$, $\mathcal{I}$).

4.10 Example

Let O , $\mathcal{N}$, $\mathcal{I}$ and f be defined as an Example 4.9. Let P = {8, 9, 10}, with P/ R= {{9}, {8, 10}} and X= {9}. Then the Nano topology $\mathcal{N}'$ = {ϕ, {9}, P} and $\mathcal{J}$ = {$\emptyset$, {9}}. Then f is ngμ-continuous but not contra $n\mathcal{I}_{g\mu}$-continuous, since f^{-1}({9})= {9} is not $n\mathcal{I}_{g\mu}$-closed in (O, $\mathcal{N}$, $\mathcal{I}$).

4.11 Remark

The composition of two contra $n\mathcal{I}_{g\mu}$-continuous maps need not be contra $n\mathcal{I}_{g\mu}$-continuous and this is shown from the following example.

4.12 Example

Let O, $\mathcal{N}$, $\mathcal{I}$, P, $\mathcal{N}'$, $\mathcal{J}$, Q, $\mathcal{N}'_*$, $\mathcal{K}$, f and g as in the Example 3.7. Clearly f and g are contra $n\mathcal{I}_{g\mu}$-continuous but their g $\circ$ f : (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (Q, $\mathcal{N}'_*$, $\mathcal{K}$) is not contra $n\mathcal{I}_{g\mu}$-continuous, because V = {1, 2} is n-open in (Q, $\mathcal{N}'_*$) but (g $\circ$ f $^{-1}$({8, 9})= f^{-1}(g^{-1}({8, 9})) = f^{-1}({8, 9}) = {8, 9}, which is not $n\mathcal{I}_{g\mu}$-closed in (O, $\mathcal{N}$, $\mathcal{I}$).

4.13 Theorem

Let f: (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (P, $\mathcal{N}'$, $\mathcal{J}$) be a map. Then the following conditions are equivalent

1. f is contra $n\mathcal{I}_{g\mu}$-continuous.
2. The inverse image of each n-open set in P is $n\mathcal{I}_{g\mu}$-closed in O.
3. The inverse image of each n-closed set in P is $n\mathcal{I}_{g\mu}$-open in O.
4. For each point o in O and each n-closed set G in P with f(o) $\in$ G, there is an $n\mathcal{I}_{g\mu}$-open set U in O containing o such that f(U) $\subset$ G.

Proof (1) $\Rightarrow$ (2). Let G be n-open in P. Then P $-$ G is n-closed in P. By definition of contra $n\mathcal{I}_{g\mu}$-continuous, f^{-1}(P $-$ G) is $n\mathcal{I}_{g\mu}$-open in O. But f^{-1}(P $-$ G) = O - f^{-1}(G). This implies f^{-1}(G) is $n\mathcal{I}_{g\mu}$-closed in O.
(2) $\Rightarrow$ (3) Let G be any n-closed set in P. Then P$-$G is n-open set in P. By the assumption of (2), f^{-1}(P $-$ G) is

$n\mathcal{I}_{g\mu}$-closed in O. But f^{-1}(P − G) = O - f^{-1}(G). This implies f^{-1}(G) is $n\mathcal{I}_{g\mu}$-open in O.
(3) ⇒ (4). Let o ∈ O and G be any n-closed set in P with f(o) ∈ G. By (3), f^{-1}(G) is $n\mathcal{I}_{g\mu}$-open in O. Set U= f^{-1}(G). Then there is an $n\mathcal{I}_{g\mu}$-open set U in O containing o such that f(U) ⊂ G.
(4) ⇒ (1). Let o ∈ O and G be any n-closed set in P with f(o) ∈ G. Then P − G is n-open in P with f(o) ∈ G. By (4), there is an $n\mathcal{I}_{g\mu}$-open set U in O containing o such that f(U) ⊂ G. This implies U= f^{-1}(G) . Therefore, O − U = O − f^{-1}(G) = f^{-1}(P−G) which is $n\mathcal{I}_{g\mu}$-closed in O. □

4.14 Theorem

Let f: (O, $\mathcal{N}$, $\mathcal{I}$) → (P, $\mathcal{N}'$, $\mathcal{J}$) and g: (P, $\mathcal{N}'$, $\mathcal{J}$) → (Q, $\mathcal{N}'_*$, $\mathcal{K}$). Then the following properties hold:

1. If f is contra $n\mathcal{I}_{g\mu}$-continuous and g is n⋆-continuous then g ∘ f is contra $n\mathcal{I}_{g\mu}$-continuous.
2. If f is contra $n\mathcal{I}_{g\mu}$-continuous and g is contra n⋆-continuous then g ∘ f is $n\mathcal{I}_{g\mu}$-continuous.
3. If f is $n\mathcal{I}_{g\mu}$-continuous and g is contra n⋆-continuous then g ∘ f is contra $n\mathcal{I}_{g\mu}$-continuous.

Proof (1) Let G be n-closed set in Q. Since g is n⋆-continuous, g^{-1}(G) is n-closed in P. Since f is contra $n\mathcal{I}_{g\mu}$-continuous, $(g \circ f)^{-1}$(G) = f^{-1}(g^{-1}(G)) is $n\mathcal{I}_{g\mu}$-open in O. Therefore g ∘ f is contra $n\mathcal{I}_{g\mu}$-continuous.
(2) Let G be any n-closed set in Q. Since g is contra n⋆-continuous, g^{-1}(G) is n-open in P. Since f is contra $n\mathcal{I}_{g\mu}$-continuous, $(g \circ f)^{-1}$(G) = f^{-1}(g^{-1}(G)) is $n\mathcal{I}_{g\mu}$-closed in O. Therefore g ∘ f is $n\mathcal{I}_{g\mu}$-continuous.
(3) Let G be any n-closed set in Q. Since g is contra n⋆-continuous, g^{-1}(G) is n-open in P. Since f is $n\mathcal{I}_{g\mu}$-continuous, $(g \circ f)^{-1}$(G) = f^{-1}(g^{-1}(G)) is $n\mathcal{I}_{g\mu}$-open in O. Therefore g ∘ f is contra $n\mathcal{I}_{g\mu}$-continuous. □

4.15 Theorem

Let f: (O, $\mathcal{N}$, $\mathcal{I}$) → (P, $\mathcal{N}'$, $\mathcal{J}$) is $n\mathcal{I}_{g\mu}$-irresolute map and g : (P, $\mathcal{N}'$, $\mathcal{J}$) → (Q, $\mathcal{N}'_*$, $\mathcal{K}$) is contra n⋆-continuous map, then g ∘ f : (O, $\mathcal{N}$, $\mathcal{I}$) → (Q, $\mathcal{N}'_*$, $\mathcal{K}$) is contra $n\mathcal{I}_{g\mu}$-continuous map.
Proof Since g is contra n⋆-continuous from (P, $\mathcal{N}'$, $\mathcal{J}$) → (Q, $\mathcal{N}'_*$, $\mathcal{K}$), for any n-open set in q as a subset of Q, we get, g^{-1}(q) = G is a n-closed set in (P, $\mathcal{N}'$, $\mathcal{J}$). By Theorem 2.11 (1) and (2), it implies that g^{-1}(q) = G is $n\mathcal{I}_{g\mu}$-closed in (P, $\mathcal{N}'$, $\mathcal{J}$). As f is $n\mathcal{I}_{g\mu}$-irresolute map. We get (g ∘ f)$^{-1}$(q)= f^{-1}(g^{-1}(q)) = f^{-1}(G) = S and S is a $n\mathcal{I}_{g\mu}$-closed in (O, $\mathcal{N}$, $\mathcal{I}$). Hence g ∘ f is a contra $n\mathcal{I}_{g\mu}$-continuous map. □

4.16 Theorem

Let f: (O, $\mathcal{N}$, $\mathcal{I}$) → (P, $\mathcal{N}'$, $\mathcal{J}$) is $n\mathcal{I}_{g\mu}$-irresolute map and g : (P, $\mathcal{N}'$, $\mathcal{J}$) → (Q, $\mathcal{N}'_*$, $\mathcal{K}$) is contra $n\mathcal{I}_{g\mu}$-continuous map, then g ∘ f : (O, $\mathcal{N}$, $\mathcal{I}$) → (Q, $\mathcal{N}'_*$, $\mathcal{K}$) is contra $n\mathcal{I}_{g\mu}$-continuous map.
Proof Since g is contra $n\mathcal{I}_{g\mu}$-continuous from (P, $\mathcal{N}'$, $\mathcal{J}$) → (Q, $\mathcal{N}'_*$, $\mathcal{K}$), for any n-open set in q as a subset of Q, we get, g^{-1}(q) = G is a $n\mathcal{I}_{g\mu}$-closed set in (P, $\mathcal{N}'$, $\mathcal{J}$). As f is $n\mathcal{I}_{g\mu}$-irresolute map. We get (g ∘ f)$^{-1}$(q)= f^{-1}(g^{-1}(q)) = f^{-1}(G) = S and S is a $n\mathcal{I}_{g\mu}$-closed in (O, $\mathcal{N}$, $\mathcal{I}$). Hence g ∘ f is a contra $n\mathcal{I}_{g\mu}$-continuous map. □

4.17 Theorem

Let f: (O, $\mathcal{N}$, $\mathcal{I}$) → (P, $\mathcal{N}'$, $\mathcal{J}$) be a map and g: (O, $\mathcal{N}$, $\mathcal{I}$) → ((O, $\mathcal{N}$, $\mathcal{I}$) × (P, $\mathcal{N}'$, $\mathcal{J}$)) the graph map of f, defined by g(o) = (o, f(o)) for every o ∈ O. If g is contra $n\mathcal{I}_{g\mu}$-continuous, then f is contra $n\mathcal{I}_{g\mu}$-continuous.

Proof Let G be an n-open set in (P, $\mathcal{N}'$, $\mathcal{J}$). Then ((O, $\mathcal{N}$, $\mathcal{I}$) × G) is an n-open set in ((O, $\mathcal{N}$, $\mathcal{I}$) × (P, $\mathcal{N}'$, $\mathcal{J}$)). It follows from Theorem 4.13, that f^{-1}(G) = g^{-1}((O, $\mathcal{N}$, $\mathcal{I}$) × G) is $n\mathcal{I}_{g\mu}$-closed in (O, $\mathcal{N}$, $\mathcal{I}$). Thus, f is contra $n\mathcal{I}_{g\mu}$-continuous. □

4.18 Theorem

If a map f : (O, $\mathcal{N}$, $\mathcal{I}$) → (P, $\mathcal{N}'$, $\mathcal{J}$) is contra $n\mathcal{I}_{g\mu}$-continuous and P is nano regular, then f is $n\mathcal{I}_{g\mu}$-continuous.

Proof Let o be an arbitrary point of O and G be any n-open set of P containiing f(o). Since P is nano regular, there exists an n-open set W in P containing f(o) such that A_n^*(W) ⊂ G. Since f is contra $n\mathcal{I}_{g\mu}$-continuous, by Theorem 4.13, there exists an $n\mathcal{I}_{g\mu}$-open set U containing o such that f(U) ⊂ A_n^*(W). Thus f(U) ⊂ A_n^*(W) ⊂ G. Hence f is $n\mathcal{I}_{g\mu}$-continuous. □

4.19 Definition

A space (O, $\mathcal{N}$, $\mathcal{I}$) is said to be an $n\mathcal{I}_{g\mu}$-space if every $n\mathcal{I}_{g\mu}$-open set is n-open in (O, $\mathcal{N}$, $\mathcal{I}$).

4.20 Theorem

A map f : (O, $\mathcal{N}$, $\mathcal{I}$) → (P, $\mathcal{N}'$, $\mathcal{J}$) is contra $n\mathcal{I}_{g\mu}$-continuous and O is $n\mathcal{I}_{g\mu}$-space, then f is contra n⋆-continuous.

Proof Let G be n-closed set in P. Since f is contra $n\mathcal{I}_{g\mu}$-continuous, f^{-1}(G) is $n\mathcal{I}_{g\mu}$-open in O. Since O is an $n\mathcal{I}_{g\mu}$-space, f^{-1}(G) is n-open in O. Therefore f is contra n⋆-continuous. □

4.21 Definition

An nano ideal topological space (O, $\mathcal{N}$, $\mathcal{I}$) is said to be $n\mathcal{I}_{g\mu}$-connected if (O, $\mathcal{N}$, $\mathcal{I}$) cannot be expressed as the union of two disjoint non empty $n\mathcal{I}_{g\mu}$-open subsets of (O, $\mathcal{N}$, $\mathcal{I}$).

4.22 Theorem

A contra $n\mathcal{I}_{g\mu}$-continuous image of a $n\mathcal{I}_{g\mu}$-connected space is nano connected.

Proof Let f : (O, $\mathcal{N}$, $\mathcal{I}$) → (P, $\mathcal{N}'$, $\mathcal{J}$) be a contra $n\mathcal{I}_{g\mu}$-continuous map of an $n\mathcal{I}_{g\mu}$-connected space (O, $\mathcal{N}$, $\mathcal{I}$) onto a nano topological space (P, $\mathcal{N}'$). If possible, let P be nano disconnected. Let G and S form a nano disconnection of P. Then G and S are nano clopen and P = G ∪ S where G ∩ S = ϕ. Since f is contra $n\mathcal{I}_{g\mu}$-continuous, O = f^{-1}(P) = f^{-1}(G ∪ S) = f^{-1}(G) ∪ f^{-1}(S), where f^{-1}(G) and f^{-1}(S) are non empty $n\mathcal{I}_{g\mu}$-open sets in O. Also f^{-1}(G) ∩ f^{-1}(S) = ϕ. Hence O is not $n\mathcal{I}_{g\mu}$-connected. This is a contradiction. Therefore P is nano connected. □

4.23 Lemma

For an nano ideal topological space (O, $\mathcal{N}$, $\mathcal{I}$), the following are equivalent.

1. O is $n\mathcal{I}_{g\mu}$-connected.

2. The only subset of O which are both $n\mathcal{I}_{g\mu}$-open and $n\mathcal{I}_{g\mu}$-closed are the empty set ϕ and O.

Proof (1) $\Rightarrow$ (2) Let G be an $n\mathcal{I}_{g\mu}$-open and $n\mathcal{I}_{g\mu}$-closed subset of O. Then O $-$ G is both $n\mathcal{I}_{g\mu}$- open and $n\mathcal{I}_{g\mu}$-closed. Since O is $n\mathcal{I}_{g\mu}$-connected, O can be expressed as union of two disjoint non empty $n\mathcal{I}_{g\mu}$-open sets O and O $-$ G, which implies O $-$ G is empty.
(2) $\Rightarrow$ (1) Suppose O $=$ G $\cup$ S where G and S are disjoint non empty $n\mathcal{I}_{g\mu}$-open subsets of O. Then G is both $n\mathcal{I}_{g\mu}$-open and $n\mathcal{I}_{g\mu}$-closed. By assumption either G $= \phi$ or O which contradicts the assumption G and S are disjoint non empty $n\mathcal{I}_{g\mu}$-open subsets of O. Therefore O is $n\mathcal{I}_{g\mu}$-connected. □

4.24 Definition

[5] A map f: (O, $\mathcal{N}$) $\rightarrow$ (P, $\mathcal{N}'$) is called nano preclosed if the image of every nano closed subset of O is nano preclosed in P.

4.25 Theorem

Let f : (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (P, $\mathcal{N}'$, $\mathcal{J}$) be a surjective nano preclosed contra $n\mathcal{I}_{g\mu}$-continuous map. If O is an $n\mathcal{I}_{g\mu}$-space, then P is nano locally indiscrete.

Proof Suppose that G is n-open in P. By hypothesis f is contra $n\mathcal{I}_{g\mu}$-continuous and therefore f^{-1}(G) $=$ U is $n\mathcal{I}_{g\mu}$-closed in O. Since O is an $n\mathcal{I}_{g\mu}$-space, U is n-closed in O. Since f is nano preclosed, then G is also nano preclosed in P. Now we have ncl(G) $=$ ncl(nint(G)) $\subset$ G. This means that G is n-closed and hence P is nano locally indiscrete. □

Conclusions In this paper is to introduce a new class of sets called $n\mathcal{I}_{g\mu}$-closed sets and $n\mathcal{I}_{g\mu}$-open sets are used to define and investigate a new class of maps called $n\mathcal{I}_{g\mu}$-continuous, $n\mathcal{I}_{g\mu}$-irresolute map and contra $n\mathcal{I}_{g\mu}$-continuous maps in nano ideal topological spaces. We discuss the relationship with some other related maps. In future, we have extend this work in various nano ideal topological field.

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