Giving Birth to Vectorial Coordinate Geometry

Pramode Ranjan Bhattacharjee

Retired Principal, Kabi Nazrul Mahavidyalaya, Sonamura, Tripura 799131, India

drpramode@rediffmail.com

Abstract

This paper deals with certain foundational questions about the adequacy of the long-running Cartesian coordinate geometry which is based on the abstract concept of "sign convention" for the study of the physical world. To establish a bridge between theory and practice, the present paper purports to introduce the "Vectorial coordinate geometry" that makes use of a modern notational system to work as an alternative for the long-running historical system. The proposed scheme will be equally applicable to the "Pure mathematical world" as well as to the "Real physical world" and is much clearer leaving no room for confusion.

Keywords: Cartesian Coordinate Geometry; Sign Convention; Cartesian Coordinates; Vector Algebra; Dot Product; Cross Product.

Introduction

We are familiar with two different branches of mathematics, Viz. (i) Pure mathematics, which is built on a firm axiomatic basis and is concerned entirely with abstract concept for study from the view point of intrinsic interest, regardless of whether there is any compliance with the real world or not, and (ii) Applied mathematics, which is concerned with the development and study of mathematical tools for dealing exclusively with the problems of the real world.

Thus there is no mandatory need for the Pure mathematical world to conform to the real world as far as measurement of quantities or their magnitudes are concerned. However, many pure mathematical tools developed so far have got unexpected applications. Descartes' geometry Loney (1895), Ashton (1905), Eisenhart (1939), itself is not merely a theoretical tool but is rich in "real world" applications. Cartesian coordinate system has been employed in Computer-aided geometric design, Computer graphics, Geometry-related data processing. Vector algebra, Matrices, Tensors, etc. are some other mathematical tools used to provide mathematical handling of Applied mathematics.

However, in selecting a pure mathematical tool to deal with a problem of the real world, care must be taken to see that such a Pure mathematical tool should not be based on any axiom/convention, which is not applicable in the real world. This fundamental fact has been overlooked so far in applying the Cartesian coordinate geometry to solve real-world problems, eg. (i) Solving typical problems of motion under gravity as mentioned in - (2008, 2011), which as per traditional technique, makes use of the sign convention of the Cartesian coordinate geometry in which height measured in the vertically upward direction is considered as positive and that measured in the vertically downward direction is taken as negative, (ii) Finding the relation(s) between object distance, image distance, and focal length of a spherical mirror (lens) making use of the Cartesian sign convention - (2002, 2012), etc. .

It may be noted that the sign convention of Cartesian coordinate geometry is not at all ambiguous so long as we are concerned with the Pure mathematical world in which, as per traditional convention, 'Distance' can be positive or negative or zero. But in the real world, only the positive 'Distance' including zero are permissible. This urges the immediate need to give birth to the vectorial coordinates and the vectorial coordinate geometry with a view to dealing with the real world problems in an unambiguous manner.

A preliminary discussion regarding the basic difference between "Coordinate", "Distance" and "Displacement" has been offered first followed by subsequent sections. Furthermore, it is felt that a preliminary idea about vector algebra and its applications by considering standard texts such as Spiegel (1990) will enhance the readability of the paper.

Revisiting the Fundamental Concepts of Coordinates, Distance and Displacement

In this section, the definitions of "Cartesian coordinate," "Distance" and "Displacement" have been revisited to have a clear picture of those three terms prevailing in the traditional literature.

In page number 8 of Ashton (1905), the author defined coordinates of a point as follows: "If instead of using the words above or below, right or left, we understand that all distances measured upward or to the right are positive, and those measured downward or to the left are negative, two numbers with the proper signs attached will represent the distances of the point from the two lines, and those two numbers taken together will locate absolutely the position of any point in the plane. These numbers, representing the distances of the point from the two lines, with their proper signs attached, are called the coordinates of the point."

In discussing Cartesian coordinates in the plane appearing at page numbers 8-9 of Eisenhart (1939), the author states: "As basis for definition of coordinates, ……… (The question of sign may be *annoying*, but in many cases it is important; there are also cases when it is not important, and the reader is expected to discriminate between these cases.)".

The concept of associating positive or negative sign to a coordinate of a point in Cartesian coordinate geometry arises from the concept of directed line segment Ashton (1905), according to which if the line AB is represented by a positive number, the line BA will be represented by the same number with a negative sign. In other words, if the length of a line generated in one direction, is represented by a positive number, then the length of a line generated in the opposite direction, is represented by a negative number.

The same type of concept regarding the measurement of length (or distance) is also available in page number 9 of Loney (1895), where the author mentioned the following quoted lines: "Line measured parallel to OX are positive whilst those measured parallel to $OX^{/}$ are negative; lines measured parallel to OY are positive and those parallel to $OY^{/}$ are negative."

It can be readily seen that the aforesaid concept of associating positive or negative sign to the length of a line segment depending upon the direction of measurement is absolutely an abstract one and it does not have any link to reality in which a length is always zero or positive irrespective of the direction of measurement.

As per traditional literature, "Distance" is a scalar quantity which implies the magnitude of length/height covered by an object during its motion. Therefore it cannot be negative.

It is worth mentioning here something about the term "Displacement" as well. In traditional literature, "Displacement" of a moving particle during a particular period of time is defined as the change in its position during that period of time and is always directed from the initial position to the final position of the particle. Thus it is clear that Displacement is a vector quantity and hence, it can be negative or positive depending on what direction is considered positive or negative from its starting point.

In view of above, unlike 'Distance' (which is a scalar quantity), "Displacement" (which is a vector quantity) can be represented by "Directed line segment."

The Cartesian Coordinate Geometry

'Coordinate geometry' is a well-known mathematical tool in which geometry is studied with the help of algebra. This branch has been introduced by Rene Descartes and is also known as 'Cartesian coordinate geometry.' Two straight lines, XX/ and YY/, known as the axes of coordinates, intersecting perpendicularly at the point O, known as the origin, define the Cartesian coordinate system. As shown in Figure 1, the distances measured along **OX** and **OY** are taken to be positive and distances measured along **OX/** and **OY/** are taken to be negative. This is the usual sign convention of coordinate geometry.

This Cartesian system (two dimensional as well as three dimensional) is extensively used in various branches of Science and Engineering. But it is to be noted that no question regarding the ambiguity of the above sign convention (which considers distance also as negative in addition to being positive) of Coordinate geometry Loney (1895), Ashton (1905), Eisenhart (1939) will arise so long as we are confined to the Pure mathematical world, which is built on firm axiomatic basis. It is well known that the Cartesian coordinate geometry has got a wide variety of real-world applications. But in applying an abstract Pure mathematical tool to solve problems in real-world, we must have to see that there exists no conflict between the axioms/conventions based on which the Pure mathematical tool works and the real-world situations. This fundamental fact has been overlooked in applying the Cartesian coordinate geometry to solve real-world problems.

'Distance', as we know, is a scalar quantity having only magnitude but no specific direction. So, it is not at all clear why a direction is being associated with 'distance' measurement in the aforesaid sign convention of Coordinate geometry. A little consideration will show that each coordinate of a point qualifies itself as a vector quantity and can best be represented by a vector rather than associating the negative sign with 'distance' measurement. Furthermore, distance in real life situation is always a non-negative quantity. So, the concept of negative distance is not in conformity with real-life situation as has been pointed out in - (2002, 2008, 2011, 2012).

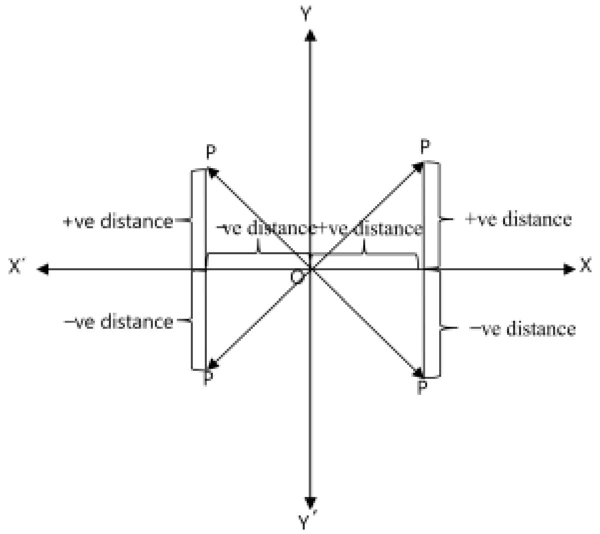

Fig. 1. Figure showing positive and negative distance as per traditional sign convention of Cartesian coordinate geometry.

Vectorial Coordinate Geometry To Emerge As A Realistic Mathematical Tool

An examination of the Cartesian system clearly reveals that each coordinate (i.e. the x-coordinate or the y-coordinate or the z-coordinate) of a point is a vector quantity. This means that to arrive at the point P, one is to move through distance of specific magnitude along specific direction (such as along x-axis or y-axis or z-axis). Each such distance in specific direction is called the Cartesian coordinate of the point P. It is, therefore, possible to give birth to the 'Vectorial coordinate geometry' that makes use of a notational system according to which coordinates get denoted as vectors.

Thus according to the new framework of 'Vectorial coordinate geometry,' a point (x, y, z) in the Cartesian coordinate system is to be represented as (x **i**, y **j**, z **k**) where i, **j** and **k** are rectangular unit vectors. Using such novel vectorial coordinates, one can now proceed to derive useful results and formulae, a few of which will be considered next.

(a) To find the distance between two points (x_1 **i**, y_1 **j**) and (x_2 **i**, y_2 **j**).

Let A(x_1 **i**, y_1 **j**) and B(x_2 **i**, y_2 **j**) be the two given points. Then, we have, **OA** = x_1 **i** + y_1 **j** and **OB** = x_2 **i** + y_2 **j**. Then considering triangle OAB, we have, by the triangle law of addition of vectors, **AB** = **OB** − **OA** = ($x_2 - x_1$) **i** + ($y_2 - y_1$) **j**. Hence, the distance between the points A and B is given by, AB = |**AB**| = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

(b) To find the coordinates of the point which divides the line segment formed by joining the points (x_1 **i**, y_1 **j**) and (x_2 **i**, y_2 **j**) internally in the ratio m : n.

Let A ≡ (x_1 **i**, y_1 **j**), B ≡ (x_2 **i**, y_2 **j**). Also, let P(x **i**, y **j**) be the point at which the line segment AB is divided internally in the ratio m: n. Now, if **s** is a unit vector along the direction of **AP** or **PB**, we have, (μm) **s** = **AP** = ($x - x_1$) **i** + ($y - y_1$) **j**, where μ is the constant of proportionality.

$$\text{or, } \mathbf{s} = \frac{\{(x - x_1)\,\mathbf{i} + (y - y_1)\,\mathbf{j}\}}{\mu m}$$

$$\text{or, } \mathbf{s} = \frac{\{(x - x_1)\,\mathbf{i}\}}{\mu m} + \frac{\{(y - y_1)\,\mathbf{j}\}}{\mu m} \quad \ldots \quad (1)$$

Again we have,

(μn) **s** = **PB** = ($x_2 - x$) **i** + ($y_2 - y$) **j**

$$\text{or, } \mathbf{s} = \frac{\{(x_2 - x)\,\mathbf{i} + (y_2 - y)\,\mathbf{j}\}}{\mu n}$$

$$\text{or, } \mathbf{s} = \frac{\{(x_2 - x)\,\mathbf{i}\}}{\mu n} + \frac{\{(y_2 - y)\,\mathbf{j}\}}{\mu n} \quad \ldots \quad (2)$$

From the relations (1) and (2) we have,

$$\frac{\{(x - x_1)\,\mathbf{i}\}}{\mu m} + \frac{\{(y - y_1)\,\mathbf{j}\}}{\mu m} = \frac{\{(x_2 - x)\,\mathbf{i}\}}{\mu n} + \frac{\{(y_2 - y)\,\mathbf{j}\}}{\mu n} \quad (3)$$

It then follows from the relation (3) that

$$\frac{\{(x - x_1)\}}{\mu m} = \frac{\{(x_2 - x)\}}{\mu n}, \text{ and } \frac{\{(y - y_1)\}}{\mu m} = \frac{\{(y_2 - y)\}}{\mu n}$$

$$\text{i.e. } x = \frac{mx_2 + nx_1}{m+n}, \text{ and } y = \frac{my_2 + ny_1}{m+n}$$

Thus the coordinates of the point which divides the line segment formed by joining the points (x_1 **i**, y_1 **j**) and (x_2 **i**, y_2 **j**) internally in the ratio m : n are $\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$.

(c) The find the area of a triangle whose vertices are (x_1 **i**, y_1 **j**), (x_2 **i**, y_2 **j**) and (x_3 **i**, y_3 **j**).

Let A ≡ (x_1 **i**, y_1 **j**), B ≡ (x_2 **i**, y_2 **j**) and C ≡ (x_3 **i**, y_3 **j**).

Now, we know that the area of triangle ABC = $\frac{1}{2}$ (AB . AC $\sin\theta$), where θ is the smaller of the angles between the vectors **AB** and **AC**.

Thus area of triangle ABC

$= \frac{1}{2} |(\mathbf{AB} \times \mathbf{AC})|$

$= \frac{1}{2} \mid [\{(x_2 - x_1)\,\mathbf{i} + (y_2 - y_1)\,\mathbf{j} + 0\,\mathbf{k}\} \times \{(x_3 - x_1)\,\mathbf{i} + (y_3 - y_1)\,\mathbf{j} + 0\,\mathbf{k}\}] \mid$

$$= \frac{1}{2} \left| \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_2 - x_1 & y_2 - y_1 & 0 \\ x_3 - x_1 & y_3 - y_1 & 0 \end{vmatrix} \right|$$

$= \frac{1}{2} \mid [(\mathbf{k}) \{(x_2 - x_1)(y_3 - y_1) - (y_2 - y_1)(x_3 - x_1)\}] \mid$

$= \frac{1}{2} |\{x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)\}|$

Thus the area of a triangle whose vertices are $(x_1\,\mathbf{i}, y_1\,\mathbf{j})$, $(x_2\,\mathbf{i}, y_2\,\mathbf{j})$ and $(x_3\,\mathbf{i}, y_3\,\mathbf{j})$ is equal to

$\frac{1}{2} |\{x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)\}|$.

The technique of finding equation of a straight line in different forms in the new frame work will now be considered.

(i) **Slope or Gradient form:**

As shown in Fig. 2, let AB be the straight line having slope 'm' so that $\tan\theta = m$. Also, let the intercept made by the straight line AB from the Y-axis be 'c' so that ON = c.

Then from figure 2, we have,

$$\tan\theta = \frac{\{(\mathbf{i} \times \mathbf{NP}) \cdot \mathbf{k}\}}{\mathbf{i} \cdot \mathbf{NP}}$$

$$= \frac{[\{\mathbf{i} \times (\mathbf{NM} + \mathbf{MP})\} \cdot \mathbf{k}]}{\{\mathbf{i} \cdot (\mathbf{NM} + \mathbf{MP})\}}$$

$$= \frac{[\{\mathbf{i} \times (x\,\mathbf{i} + y\,\mathbf{j} - c\,\mathbf{j})\} \cdot \mathbf{k}]}{\{\mathbf{i} \cdot (x\,\mathbf{i} + y\,\mathbf{j} - c\,\mathbf{j})\}}$$

$$= \frac{y - c}{x}$$

$$\text{or, } m = \frac{y - c}{x}$$

Hence the equation of the straight line in slope or gradient form is given by, $y = m\,x + c$

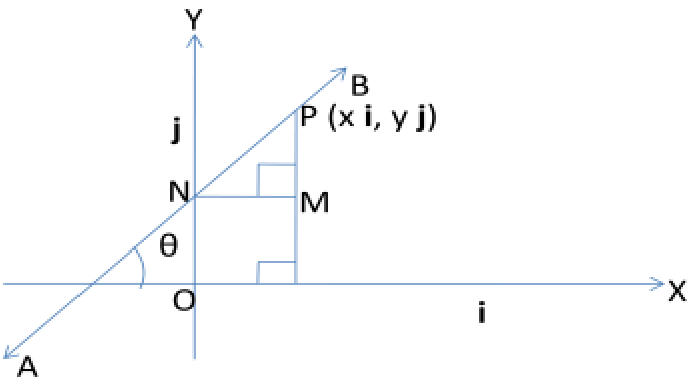

Fig. 2. Diagram for finding the equation of a straight line in slope or gradient form.

(ii) **Slope-point form:**

As shown in Fig. 3, let the straight line AB passing through the given point Q(x_1 **i**, y_1 **j**) be of slope 'm'. Also, let us consider any point P(x **i**, y **j**) on the straight line AB. Then considering Figure 3, we have,

$$m = \tan\theta = \frac{\{(\mathbf{i} \times \mathbf{PQ})\cdot \mathbf{k}\}}{\mathbf{i}\cdot \mathbf{PQ}}$$

$$= \frac{[\{\mathbf{i} \times (\mathbf{PM} + \mathbf{MQ})\}\cdot \mathbf{k}]}{\{\mathbf{i}\cdot(\mathbf{PM} + \mathbf{MQ})\}}$$

$$= \frac{[\{\mathbf{i} \times (x_1\mathbf{i} - x\,\mathbf{i} + y_1\,\mathbf{j} - y\,\mathbf{j})\}\cdot \mathbf{k}]}{\{\mathbf{i}\cdot(x_1\mathbf{i} - x\,\mathbf{i} + y_1\mathbf{j} - y\,\mathbf{j})\}}$$

$$= \frac{y_1 - y}{x_1 - x}$$

Hence the equation of the straight line in slope-point form is given by,

$$y - y_1 = m\,(x - x_1)$$

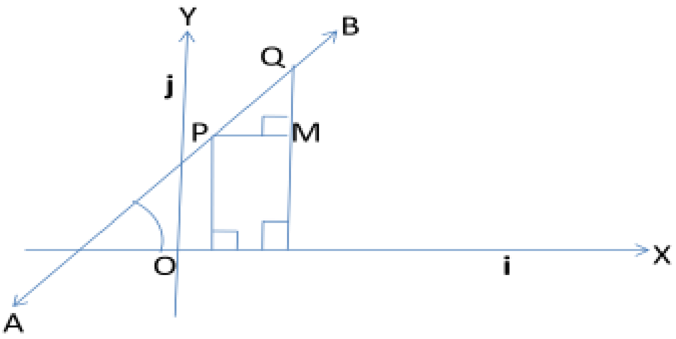

Fig. 3. Diagram for finding the equation of a straight line in slope point form.

(iii) **Intercept form:**

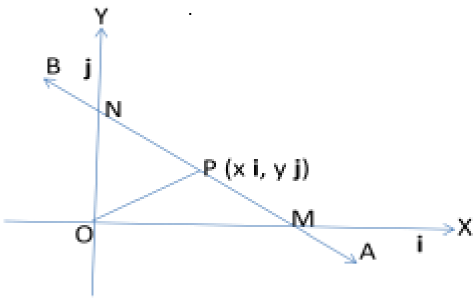

Fig. 4. Diagram for finding the equation of a straight line in intercept form.

As shown in Fig. 4, let the intercepts made by the straight line AB from the axes of coordinates be respectively 'a' and 'b' so that OM = a, and ON = b.

Now since the points M, P and N are collinear, we have,

$$\mathbf{MP} \times \mathbf{PN} = \mathbf{0}$$

$$\text{or, } (\mathbf{OP} - \mathbf{OM}) \times (\mathbf{ON} - \mathbf{OP}) = \mathbf{0}$$

$$\text{or, } (x\,\mathbf{i} + y\,\mathbf{j} - a\,\mathbf{i}) \times (b\,\mathbf{j} - x\,\mathbf{i} - y\,\mathbf{j}) = \mathbf{0}$$

$$\text{or, } xy + (x - a)(b - y) = 0$$

or, $\frac{x}{a} + \frac{y}{b} = 1$, which is the required equation of the straight line in intercept form.

(iv) Perpendicular form:

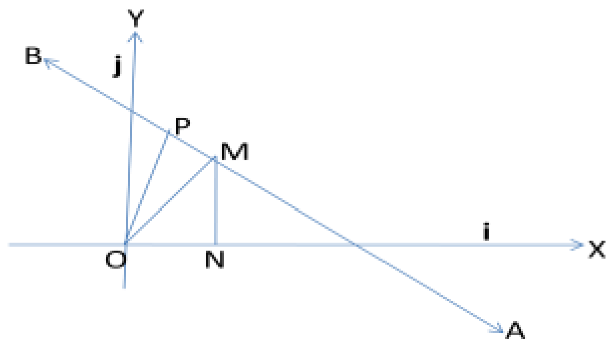

Fig. 5. Diagram for finding the equation of a straight line in perpendicular form.

Let 's' be the length of the perpendicular dropped from the origin on to the required straight line AB as shown in Figure 5. Also, let θ be the angle which this perpendicular OM makes with the X axis. Then considering Figure 5, if M ≡ (x_1 **i**, y_1 **j**), we get,

$$x_1 = s \cos\theta \quad \text{and} \quad y_1 = s \sin\theta.$$

Then the vectorial coordinates of M are (s cos θ **i**, s sin θ **j**)

Let P (x **i**, y **j**) be any point on the straight line AB.

Hence, **MP** = **OP** – **OM** = (x – s cos θ) **i** + (y – s sin θ) **j**

Now, **OM** = s cos θ **i** + s sin θ **j**

Since **OM** is perpendicular to **MP**, we have **OM** · **MP** = 0

or, (s cos θ **i** + s sin θ **j**)·{(x – s cos θ) **i** + (y – s sin θ) **j**} = 0

or, x cos θ + y sin θ = s, which is the required equation of the straight line in perpendicular form.

Finally, the technique of derivation of the perpendicular distance of a given straight line from a given point in the new framework is being considered.

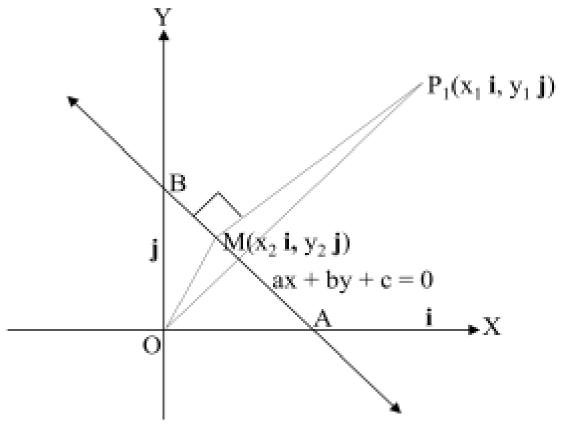

Fig. 6. Diagram for determining perpendicular distance of the straight line ax + by + c = 0 from the point (x_1 i, y_1 j).

With reference to Figure 6, we have,

$$\mathbf{OA} = -\frac{c}{a}\mathbf{i} \quad \text{and} \quad \mathbf{OB} = -\frac{c}{b}\mathbf{j}$$

Then, **AB** = **OB** – **OA**

$$= \left(\frac{c}{a}\right)\mathbf{i} - \left(\frac{c}{b}\right)\mathbf{j}$$

Let P_1M be dropped perpendicular to the given straight line AB from the given point

$P_1(x_1\,\mathbf{i}, y_1\,\mathbf{j})$. Also, let the vectorial coordinates of M be $(x_2\,\mathbf{i}, y_2\,\mathbf{j})$. Then, we have,

$$\mathbf{P_1M} = \mathbf{OM} - \mathbf{OP_1} = (x_2 - x_1)\,\mathbf{i} + (y_2 - y_1)\,\mathbf{j}$$

Now, since $\mathbf{P_1M}$ is perpendicular to **AB**, we have,

$$\mathbf{P_1M} \cdot \mathbf{AB} = 0$$

$$\text{or, } \{(x_2 - x_1)\,\mathbf{i} + (y_2 - y_1)\,\mathbf{j}\} \cdot \{\frac{c}{a}\,\mathbf{i} - \frac{c}{b}\,\mathbf{j}\} = 0$$

$$\text{or, } bcx_2 - acy_2 = bcx_1 - acy_1 \qquad \dots \qquad (4)$$

Again the point M $(x_2\,\mathbf{i}, y_2\,\mathbf{j})$ lies on the straight line ax + by + c = 0

$$\text{Hence, } ax_2 + by_2 = -c \qquad \dots \qquad (5)$$

Equations (4) and (5) can now be solved for x_2 and y_2 to get

$$x_2 = \frac{b^2x_1 - aby_1 - ac}{a^2+b^2}, \text{ and } y_2 = \frac{-abx_1 + a^2y_1 - bc}{a^2+b^2}$$

Hence the required perpendicular distance

$= P_1M = \sqrt{\{(x_2 - x_1)^2 + (y_2 - y_1)^2\}}$

$= \sqrt{\{(\frac{b^2x_1 - aby_1 - ac}{a^2+b^2} - x_1)^2 + (\frac{-abx_1 + a^2y_1 - bc}{a^2+b^2} - y_1)^2\}}$

$= +\frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}}$ or, $-\frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}}$,

where the positive or negative sign is to be taken depending on whether the quantity

(ax_1+by_1+c) is positive or negative.

Some activities for investigating the approach offered

In this section, some more activities are being provided along with hints at some places so as to allow the readers investigate the novel approach by themselves.

Activity 1. Use the relation, $\tan\theta = \frac{\{(\mathbf{i} \times \mathbf{AB}) \cdot \mathbf{k}\}}{\mathbf{i} \cdot \mathbf{AB}}$, to find the slope or gradient of the straight line passing through the two given points A(x_1 **i**, y_1 **j**) and B(x_2 **i**, y_2 **j**).

Activity 2. Show that the points A(0 **i**, 0 **j**), B(**i**, – 2 **j**), and C(– **i** , 2 **j**) are collinear.

Hints: Make use of the cross product of any two of the vectors **AB**, **BC**, and **CA** for the said purpose.

Activity 3. Obtain the equation of a straight line passing through two given points A(x_1 **i**, y_1 **j**) and B(x_2 **i**, y_2 **j**).

Hints: Taking any point P(x **i**, y **j**) on the straight line passing through the two given points A(x_1 **i**, y_1 **j**) and B(x_2 **i**, y_2 **j**), the relation **AB** × **AP** = **0**, may be used for the desired purpose.

Activity 4. Find the equation of a circle having centre at the point C(α **i**, β **j**) and radius equal to 'a'.

Hints: Considering any point P(x **i**, y **j**) on the circumference of the circle, the required equation can be obtained by equating the modulus of **CP** to 'a'.

Activity 5. Find the equation of a circle whose diameter is the line segment joining the points A(x_1 **i**, y_1 **j**) and B(x_2 **i**, y_2 **j**).

Hints: Since the angle subtended by the diameter of a circle at any point on its circumference is 90^0, the desired equation can be found out by considering any point P(x **i**, y **j**) on the circumference of the circle, and making use of the relation, **PA** · **PB** = 0.

Conclusion

The concept of negative distance or the so-called 'signed distance' or 'directed distance' in Cartesian coordinate geometry is an abstract one and such an abstract concept of negative distance could be taken to be granted so long as we are confined to the pure mathematical world, which is built on a firm axiomatic basis. But this type of concept of negative distance has no place in reality. Real world could only accept positive value of distance (including the value zero), which is exclusively a scalar quantity. Thus considering the real world, the present paper purports to introduce the "Vectorial coordinate geometry" to work as an alternative for the well-known historical system which has been in constant use all over the world for many years, the differences between the two being essentially philosophical on account of the following reasons.

(i) Unlike the long-established Cartesian system, the present scheme makes use of a realistic notational system according to which coordinates themselves get denoted as vectors.

(ii) As per relevant sign convention, the positive or negative sign of a coordinate in Cartesian coordinate geometry is closely associated with the direction of measurement of distance. For example, if the Cartesian coordinates of a point P are (–2, 4), then to arrive at the point P, one has to move 2 units of distance from the origin along the negative direction of X-axis, and 4 units of distance from the origin along the positive direction of Y-axis.

On the other hand, the positive or negative sign of a coordinate alone in the vectorial representation is not at all associated with the direction of measurement of distance. For example, if the vectorial coordinates of a point P are (–2 **i**, 4 **j**), then it implies that to arrive at the point P, one has to move 2 units of distance from the origin along the direction of – **i**, and 4 units of distance from the origin along the direction of **j**.

(iii) When we set up a correspondence between points in the plane and pairs of numbers, we do not write explicitly the basis vectors of this coordinate system, but leave them implicit. The present proposal of Vectorial coordinate geometry considers writing them explicitly, replacing (a, b) with (a **i**, b **j**).

(iv) While Cartesian coordinate geometry is based on the "ordinary scalar algebra", the proposed scheme of Vectorial coordinate geometry is based on the sophisticated mathematical tool "Vector algebra".

(v) Unlike the traditional Cartesian coordinate geometry, the present scheme increases the range of applicability of vector algebra.

The inherent Vectorial nature of the coordinate of a point has been employed to give birth to the 'Vectorial coordinate geometry'. In the new framework, each coordinate of a point is to be treated as a vector quantity unlike the traditional scheme of representation of coordinates of a point in the long-used coordinate geometry. Furthermore, greater clarity would be achieved from being explicit in the notations of coordinates as have been proposed for the new system. For example, in some calculations, we may have to switch from one basis to another. Thus for example, when solving a problem involving a ball rolling down an inclined plane, we might switch from the x-y basis to one using vectors parallel and perpendicular to that plane. In such a case, a lack of clarity can lead to confusion over what a given pair of numbers (x, y) is meant to mean, and so a more explicit notation could be useful.

The novel theoretical tool offered in this paper will enrich, sophisticate as well as enhance the relevant field of study. It is felt that there is an urgent need of recognition of this mathematical tool that makes use of a modern notational system as an alternative for the historical system for deepening thinking about Coordinate geometry and bringing a rationality between theory and practice.

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