Information Dynamic Analyses on American Football

Takeo R.M. Nakagawa, and Ai Nakagawa

Academy of Hakusan, 2-14, Meiko, Hakusan 920-2152 Japan

npo.hakusan@kjc.biglobe.ne.jp

Abstract

This paper is concerned with the information dynamic analyses on American football games: One is the game in Eastern University American Football League, National Defense Academy vs. Chiba University, and the other is the Rice Bowl (Japan Championship)2020 Final, Fujitsu vs. Kansai University. The former game is so called one-sided one, for National Defense Academy keeps the advantageous position against Chiba University throughout the game except at the normalized time $\eta=0.25$. In the range that the normalized time $\eta$ is greater than the value of 0.75, the model curve of $\xi = \eta^7$ shows good agreement with game data. It is found that the game point is at the normalized time $\eta \approx 0.9$, where the game outcome becomes definitive and is the cross point between the certainty of game outcome $\xi$ and the uncertainty of game outcome $\varsigma$. The latter game modelled by $\xi = \eta^{1.5}$ is typical one-sided one, and the game point is at the normalized time $\eta=0.62$. The two characteristic points in American football become evident through the present work, viz. (a) rules are rational and well organized, though sometimes they are too complicated, and (b) anyone can easily participate in the game not only as player, but supervisor, coach, cheer girl or member of cheering party, and/or supporter.

Keywords: American Football, Advantage, Certainty of Game Outcome, Game Point, Rational Rule, Rice Bowl, Game Pattern

1. Introduction

American football, referred to simply as ‘football’ in the United States and Canada [3], is team sports played by two teams of 11 players on a rectangular field (ca. 109.73 m long, 48.76 m wide) with goalposts (width between two posts=5.64 m and height of the cross bar=3.05 m) at each end. The offense, the team with possession of the oval-shaped football (length=ca.27.6~29.1 cm, longest outer circumferencial length=70.5~72.4 cm, and shortest outer circumferencial length=52.7~54.0 cm, weight=397~425 g, air pressure $p=703~949\text{g/cm}^2$), attempts to advance down the field by running with the ball or passing it, while the defense, the team without possession of the ball, aims to stop the offense’s advance and to take control of the ball for themselves [4]. The offense must advance at least ten yards in four downs or plays; if they fail, they turn over the football to the defense, but if they succeed, they are given a new set of four downs to continue the drive. Points are scored primarily by advancing the ball into the opposing team’s end zone for a touchdown(6 points) or kicking the ball flying through between the opponent’s two goalposts and over the cross bar for a field goal(1 or 3 points). The team wins by taking the greater points than the opponent team at the end of a game.
Game may be defined as an virtual creature consisting of start, play and end. It is, therefore, considered that even annual ring of a tree, construction of building, together with Soccer[7], Chess, Shogi[11], Judo[8], baseball[9], rugby[10] or any life of living things is a kind of games, in which methodology for modelling, analyses, prediction of the outcome is common in many respects.

Knowledge about game designs and play patterns have grown fairly well, but little advancement has been made to clarify game history, which denotes how information of game varies with the time. Currently the information dynamic model [5], in which simplicity and generality are characteristics, only makes it possible to treat and identify game history such as advantage and/or certainty of game outcome depending on the time. As far as the author is aware of, no other existing model is useful, for the applicability is severely limited due to too much sophistication or lack of generality.

The main purpose of the present study is to analyze two American football games, National Defense Academy vs. Chiba University of Eastern University American Football League, Japan, and Fujitsu vs. Kansai University of the Rice Bowl(Japan Championship) 2020 Final, in terms of information dynamic model to reflect the games critically and to prepare for future game.

2. Method of Analyses

Elemental procedure for obtaining the advantage $\alpha$, certainty of game outcome $\xi$, and uncertainty of game outcome $\zeta$ will be explained by using soccer game between teams A and B, where only goal(s) is treated as the evaluation function score for clarity.

The advantage $\alpha$ is defined as follows: When the total scores of the two teams at the end of game $S_T \neq 0$,

$$\alpha = \frac{S_A(\eta) - S_B(\eta)}{S_T} \text{ for } 0 \leq \eta \leq 1,$$  

where $S_A(\eta)$ is the current goal sum for team A(winner), $S_B(\eta)$ is the current goal sum for team B(loser), and $\eta$ is the normalized time, which is normalized by the total time of the game.

When $\alpha > 0$, team A (winner) gets the advantage against team B (loser) in the game, while when $\alpha < 0$, team B (loser) gets the advantage against team A (winner). It is certain that when $\alpha = 0$ the game is balanced. When the total scores of the two teams at the end of game $S_T = 0$, $\alpha = 0$ for $0 \leq \eta \leq 1$.

The certainty of game outcome means what extent the game outcome (i.e., win or loss) is certain depending on the time during the game, and the extent is given by the normalized value ranging from 0 to 1. The certainty of game outcome $\xi$ during the game is defined as follows: When the total scores of the two teams at the end of game $S_T \neq 0$,

$$\xi = \frac{|S_A(\eta) - S_B(\eta)|}{S_T} \text{ for } 0 \leq \eta < 1$$

=1(completed game) for $\eta=1$ or
\( \xi = 0 \) (drawn game) for \( \eta = 1 \).

At \( \eta = 1 \), \( \xi \) is assigned to the value of 1, for at the end of completed game the information on the game outcome must be full certainty. Note that \( \xi \) takes always positive value, for it is no more than the absolute value of advantage \( \alpha \) except at \( \eta = 1 \). The reason why we take the absolute value of advantage \( \alpha \) to get certainty of game outcome \( \xi \) for \( 0 \leq \eta < 1 \) is that certainty of game outcome \( \xi \) is independent of the sign of advantage \( \alpha \): As the absolute value of advantage \( \alpha \) increases (decreases), certainty of game outcome \( \xi \) must increase (decrease). On one hand, in case of drawn game, certainty of game outcome \( \xi \) may be assigned to the value of 0 at the end of game \( \eta = 1 \), for the game is right back where it starts. When the total scores of the two teams at the end of game \( S_T = 0 \), \( \xi = 0 \) for \( 0 \leq \eta \leq 1 \).

The uncertainty of game outcome \( \zeta \) during the game is defined as follows

\[
\zeta = 1 - \xi. \tag{3}
\]

Keeping in mind the foregoing elemental procedure to obtain the advantage \( \alpha \), and certainty of game outcome \( \xi \), it is straightforward to apply them to actual American football. In American football, it may be evident that points by touchdown, and/or field goal are evaluation function scores, for these are critical factors for the game. In computer shogi, evaluation function score for each of the piece moves has been assessed by a human similarly to that in computer chess based on objective knowledge and experiences [11]. Since accuracy of the evaluation function score for each of the piece moves has been improved by repeating trial and error in such a level that Bonkiras (Computer Shogi World Champion in 2012) beats Kunio Yonenaga (Shogi Champion or Meijin). Today, evaluation function score for each of the piece moves in popular board games such as Chess, Shogi and/or Go can be calculated by using computer soft with sufficient accuracy.

2. Case Studies

2.1 National Defense Academy vs. Chiba University

In this section, American Football, National Defense Academy vs. Chiba University has been analyzed by the information dynamic method of analysis.

Shown is the points scored during the game in Table 1.

Table 1 Point record of National Defense Academy vs. Chiba University

<table>
<thead>
<tr>
<th>Quarter</th>
<th>National Defense Academy</th>
<th>Chiba University</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>Total</td>
<td>23</td>
<td>9</td>
</tr>
</tbody>
</table>
After Izeki[6]

The brief history of the game, National Defense Academy (NDA) vs. Chiba University (Chiba U.) has been explained, where duration for one quarter is of 12 minutes.

• First Quarter: At 1 minute 29 seconds, NDA makes “off-tackle run” from 30yd, and then no.16, Ito succeeds touchdown of 6 points, but fails field goal (FG) for the bonus point. At 11 minutes 7 seconds, Chiba U. conducts “jet motion” from 7yd, where outside back (OB), no.4, Maekawa moves quickly towards QB, and receives the ball from QB, and then Maekawa dodges the opponent defense players, and finally succeeds in touchdown of 6 points with failure of field goal (FG) for the bonus point.

• Second Quarter: At 8 minutes 39 seconds, SB of NDA makes reverse: FB who receives the ball from QB runs to SB direction, while SB, no.14, Matsuo, who is located between T and WR, receives the ball from FB running to the opposite direction of FB, continues to run and makes touchdown of 6 points, and then succeeds field goal (FG) for the bonus point of 1.

• Third Quarter: At 6 minutes 28 seconds, Chiba U. gets field goal (FG) of 3 points from 33yd.

• Fourth Quarter: At 6 minutes 25 seconds, no.12, Tanioka of NDA succeeds field goal (FG) of 3 points from 20yd. At 8 minutes 7 seconds, no.22, Osaki, FB of NDA makes touchdown of 6 points from 33yd by “dive”, which means that FB receives the ball from QB, and then runs into the central area of the opponent field. After the touchdown, the bonus point of 1 is added to NDA by the success of field goal (FG). As the result, National Defense Academy beats Chiba University by the score 23 against 9.
Table 2 Results of data analysis for National Defense Academy vs. Chiba University

<table>
<thead>
<tr>
<th>η</th>
<th>α</th>
<th>ξ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.125</td>
<td>0.1875</td>
<td>0.1875</td>
</tr>
<tr>
<td>0.25</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.325</td>
<td>0.219</td>
<td>0.219</td>
</tr>
<tr>
<td>0.5</td>
<td>0.219</td>
<td>0.219</td>
</tr>
<tr>
<td>0.625</td>
<td>0.219</td>
<td>0.219</td>
</tr>
<tr>
<td>0.75</td>
<td>0.125</td>
<td>0.125</td>
</tr>
<tr>
<td>0.875</td>
<td>0.4375</td>
<td>0.4375</td>
</tr>
<tr>
<td>1</td>
<td>0.4375</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 2 shows the results of the present analysis, where the total time is 48, and total score points are of 32.

Figure 2 shows the relation between the advantage α and the normalized game length η. It may be clear that this game is categorized as ‘one sided game’ of National Defense Academy (NDA), for NDA keeps the advantageous position against Chiba University throughout the game except at η=0.25[12].
2.2 Fujitsu vs. Kansai University

In this section, Rice Bowl 2020 Final, Fujitsu vs. Kansai University has been analyzed by the information dynamic method of analysis (Figure 4).

Figure 3 Certainty of game outcome $\xi$ against normalized game length $\eta$.

Figure 3 shows the relation between the certainty of game outcome $\xi$ and the normalized game length $\eta$.

Figure 4 Rice Bowl 2020 Final, Fujitsu vs. Kansai University. Fujitsu WR, Nakamura succeeds touchdown in the first quarter. After Asahi [2].
Shown is the points scored during the game in Table 3.

<table>
<thead>
<tr>
<th>Quarter</th>
<th>Fujitsu</th>
<th>Kansai University</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>14</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>38</strong></td>
<td><strong>14</strong></td>
</tr>
</tbody>
</table>

The brief history of the game, Fujitsu vs. Kansai University has been explained, where duration for one quarter is 15 minutes.

- First quarter: At 6 minutes 37 seconds, Fujitsu QB, Takagi throws 10 yards pass to Kyo who succeeds touchdown of 6 points, and Nishimura gets field goal (FG) of 1 point for the bonus point. At 10 minutes 10 seconds, Fujitsu QB, Takagi makes 26 yards pass to WR, Nakamura, who succeeds touchdown of 6 points, and Nishimura gets field goal (FG) of 1 point for the bonus point.

- Second Quarter: At 7 minutes 29 seconds, Fujitsu QB, Takagi makes 23 yards pass to Iwamatsu, who succeeds touchdown of 6 points, and then Nishimura makes success of field goal (FG) as the bonus point of 1. At 11 minutes 43 seconds, Kansai University, Miake succeeds touchdown after 64 yards run, and Ando gets field goal (FG) of 1 point as the bonus point. At 13 minutes 50 seconds, Fujitsu RB, Grant makes 41 yards run, and then succeeds touchdown of 6 points, and the bonus point of 1 is added.

- Third Quarter: At 3 minutes 27 seconds, Fujitsu Nishimura gets field goal (FG) of 3 points by kicking the ball for the distance of 32 yards.

- Fourth Quarter: At 1 minute 17 seconds, Fujitsu Kin succeeds touchdown after 5 yards run, and adds the bonus point of 1 by the field goal (FG). At 14 minutes 22 seconds, Kansai University, Okuno throws 4 yards pass to Suzuki, who succeeds touchdown of 6 points and Ando gets the bonus point of 1 by field goal (FG). As the result, Fujitsu beats Kansai University by the score 38 against 14.
Table 4 Results of data analyses for Fujitsu vs. Kansai University

<table>
<thead>
<tr>
<th>η</th>
<th>α</th>
<th>ξ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.1</td>
<td>0.13</td>
<td>0.13</td>
</tr>
<tr>
<td>0.17</td>
<td>0.27</td>
<td>0.27</td>
</tr>
<tr>
<td>0.37</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>0.45</td>
<td>0.27</td>
<td>0.27</td>
</tr>
<tr>
<td>0.48</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>0.56</td>
<td>0.46</td>
<td>0.46</td>
</tr>
<tr>
<td>0.77</td>
<td>0.6</td>
<td>0.6</td>
</tr>
<tr>
<td>0.99</td>
<td>0.46</td>
<td>0.46</td>
</tr>
<tr>
<td>1</td>
<td>0.46</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 4 shows the results of the present analysis, where the total time is 60, and total score points are of 52.

Figure 5 shows the relation between the advantage α and the normalized time η. It may be clear that this game is categorized as 'one sided game' of Fujitsu, for Fujitsu keeps the advantageous position against Kansai University throughout the game.
Figure 6 Certainty of game outcome $\xi$ against normalized time $\eta$.

Figure 6 shows the relation between the certainty of game outcome $\xi$ and the normalized time $\eta$.

5. Discussion

In this section, the game point, at which the game outcome becomes definitive, will be obtained.

Figure 7 Certainty of game outcome $\xi$ against normalized time $\eta$.

Figure 7 shows the relation between the certainty of game outcome $\xi$ and the normalized time $\eta$, where the model curves $\xi=\eta^3$, $\eta^5$, and $\eta^7$ are plotted concurrently. For the more details about the information dynamic model, refer Nakagawa & Nakagawa[7]. It may be notable that after $\eta=0.75$ model curve $\xi=\eta^7$ shows good
agreement with the data, so that this curve has been selected for obtaining the game point. This is because only the curve $\xi$ passing through data being greater than the value of 0.5 is entitled to fix the game point.

![Figure 8](image_url)

**Figure 8 Information of game outcome against normalized time $\eta$.**

Figure 8 shows the relation between the information and the normalized time $\eta$, where the two curves of certainty of game outcome $\xi = \eta^7$ and the uncertainty of game outcome $\varsigma = 1 - \eta^7$, respectively, are plotted concurrently. It is found that these two curves cross with each other at $\eta \approx 0.9$. This cross point is no more than the game point, where the game outcome becomes definitive. In other words, once this game passes the game point, the game outcome cannot be reversed at any rate.

![Figure 9](image_url)

**Figure 9 Certainty of game outcome $\xi$ against normalized time $\eta$.**
Figure 9 shows the relation between the certainty of game outcome $\xi$ and the normalized time $\eta$, where the model curves $\xi = \eta, \eta^1.5$, and $\eta^2$ are plotted concurrently. It may be notable that $\xi = \eta^{1.5}$ shows reasonable agreement with the data through the game, so that this curve has been selected for obtaining the game point.

![Figure 9](image)

**Figure 9** The relation between the certainty of game outcome $\xi$ and the normalized time $\eta$.

Figure 10 shows the relation between the information and the normalized time $\eta$, where the two curves of certainty of game outcome $\xi = \eta^{1.5}$ and the uncertainty of game outcome $\eta = 1 - \eta^{1.5}$, respectively, are plotted concurrently. It is found that these two curves cross with each other at $\eta \approx 0.62$. This cross point is no more than the game point, where the game outcome becomes definitive.

**Figure 10** Information of game outcome against normalized game length $\eta$.

6. Conclusions

In this section, the new knowledge and insights obtained through the present study are summarized;

1. The two games studied are typical ‘one-sided game’; for National Defense Academy keeps the advantageous position against Chiba University throughout the game except at the normalized time $\eta = 0.25$, and Fujitsu keeps the advantage position against Kansai University throughout the game.

2. In the range that the normalized time $\eta$ is greater than the value of 0.75, the model curve of $\xi = \eta^1$ for National Defense Academy vs Chiba University shows good agreement with game data, while the model curve of $\xi = \eta^{1.5}$ for Fujitsu vs. Kansai University fits with game data reasonably well.
3. It is found that the game point of National Defense Academy vs. Chiba University is at the normalized time \( \eta \approx 0.9 \), where the game outcome becomes definitive and is no more than the cross point between the certainty of game outcome \( \xi \) and the uncertainty of game outcome \( \varsigma \). On one hand, the game point of Fujitsu vs. Kansai University is at the normalized time \( \eta \approx 0.62 \).

4. Recommended is systematic study to seek for the reason why American football is so popular in USA, but not in other countries. Two characteristic points however become evident through the present work, viz. (a) rules are rational and well organized, though sometimes they are too complicated, and (b) anyone can easily participate in the game not only as player, but supervisor, coach, cheer girl or member of cheering party, and/or supporter.

7. Acknowledgements

The present authors are grateful to Mr. Tesuzo Izeki of American Football Club, National Defense Academy, Japan for providing us the precious data on the game between National Defense Academy vs. Chiba University together with various advices on interpreting rules and/or data.

References

Appendix: Offensive and Defensive Units

The role of the offensive unit is to advance the football down the field with the ultimate goal of scoring a touchdown.

Figure 6 A diagram of a typical pre-snap formation.

After American football-Wikipedia [1]

The offense (red) is lined up in variation of the one formation, while the defense (blue) is lined up in the 4–3 defense. Both formations are legal.

The offensive team must line up in a legal formation before they can snap the ball. An offensive formation is considered illegal if there are more than four players in the backfield or fewer than five players on the offensive line. Players can line up temporarily in a position whose eligibility is different from what their number permits as long as they report the change immediately to the referee, who then informs the defensive team of the change. Neither team's players, except the center (C), are allowed to line up in or cross the neutral zone until the ball is snapped. Interior offensive linemen are not allowed to move until the snap of the ball.

The main backfield positions are the quarterback (QB), halfback/tailback (HB/TB) and fullback (FB). The quarterback(QB) is the leader of the offense. Either the quarterback(QB) or a coach calls the plays. Quarterbacks typically inform the rest of the offense of the play in the huddle before the team lines up. The quarterback(QB) lines up behind the center(C) to take the snap and then hands the ball off, throws it or runs with it.

The primary role of the halfback(HB), also known as the running back or tailback(TB), is to carry the ball on running plays. Halfbacks(HB) may also serve as receivers. Fullbacks(FB) tend to be larger than halfbacks(HB) and function primarily as blockers, but they are sometimes used as runners in short-yardage situations and are seldom used in passing situations.

The offensive line (OL) consists of several players whose primary function is to block members of the defensive line from tackling the ball carrier on running plays or sacking the quarterback(QB) on passing plays. The leader
of the offensive line is the center(C), who is responsible for snapping the ball to the quarterback(QB), blocking, and for making sure that the other linemen do their jobs during the play. On either side of the center are the guards(G), while tackles(T) line up outside the guards.

The principal receivers are the wide receivers(WR) and the tight ends(TE). Wide receivers(WR) line up on or near the line of scrimmage, split outside the line. The main goal of the wide receiver(WR) is to catch passes thrown by the quarterback(QB), but they may also function as decoys or as blockers during running plays. Tight ends(TE) line up outside the tackles(T) and function both as receivers and as blockers.

Defensive Unit

The role of the defense is to prevent the offense from scoring by tackling the ball carrier or by forcing turnovers (interceptions or fumbles).

The defensive line(DL) consists of defensive ends(DE) and defensive tackles(DT). Defensive ends(DE) line up on the ends of the line, while defensive tackles(DT) line up inside, between the two defensive ends(DE). The primary responsibilities of defensive ends(DE) and defensive tackles(DT) are to stop running plays on the outside and inside, respectively, to pressure the quarterback(QB) on passing plays, and to occupy the line so that the linebackers(LB) can break through.

Linebackers(LB) line up behind the defensive line but in front of the defensive backfield. They are divided into two types: middle linebackers(MLB) and outside linebackers(OLB). Linebackers(LB) are the defensive leaders and call the defensive plays. Their diverse roles include defending the run, pressuring the quarterback(QB), and guarding backs, wide receivers(WR) and tight ends(TE) in the passing game.

The defensive backfield, often called the secondary, consists of Cornerbacks(CB) and safeties(S). Safeties(S) are themselves divided into free safeties(FS) and strong safeties(SS). Cornerbacks(CB) line up outside the defensive formation, typically opposite a receiver to be able to cover them. Safeties(S) line up between the cornerbacks(CB) but farther back in the secondary. Safeties(S) are the last line of defense and are responsible for stopping deep passing plays as well as running plays.

There are two main ways the offense can advance the ball: running and passing. In a typical play, the center(C) passes the ball backwards and between their legs to the quarterback(QB) in a process known as the ‘snap’. The quarterback(QB) then either hands the ball off to a back, throws the ball, or runs with it. The play ends when the player with the ball is tackled or goes out-of-bounds or a pass hits the ground without a player having caught it. A forward pass can be legally attempted only if the passer is behind the line of scrimmage; only one ‘forward pass’ can be attempted per down. Like in rugby, players can also pass the ball backwards at any point during a play.

The offense is given a series of four plays, known as downs. If the offense advances ten or more yards in the four downs, they are awarded a new set of four downs. If they fail to advance ten yards, possession of the
football is turned over to the defense. In most situations, if the offense reaches their fourth down they will punt the ball to the other team, which forces them to begin their drive from farther down the field; if they are in field goal range, they might attempt to score a field goal (FG) instead.