# Non-interacting spin systems described by probabilistic soft sets

Amir Pishkoo[1], Yosra Sharifi[2], Ali Khotanloo[2],

[1]Physics and Accelerators Research School, Nuclear Science and Technology Research Institute, P.O.Box 14395-836, Tehran, Iran
[2]Faculty of Mathematical Sciences and Statistics, Malayer University

e-mail:apishkoo@gmail.com, yosra_sharifi@yahoo.com, a.khotanloo@malayeru.ac.ir

## Abstract

In this paper we study the system of non-interacting spins like electrons by using notions probability, soft sets and probabilistic soft sets. Electrons are indistinguishable identical particles whose spin is one of their features like mass of electron, charge of electron etc. We also compare two systems of non-interacting spins by defining two soft spin sets and calculating their similarity measure.

**Keywords:** Soft set, Probability, Non-interacting spins, Probabilistic soft set.

## Introduction

Probability appeared as a logical and reasonable notion in western culture around 1650 and theory of probability originated from the Pascal-Fermat correspondence of 1654 in history of mathematics [1]. This theory is built upon a fundamental set of equally probable outcomes [2]. Although scientific laws such as Newton's laws were absolute in the 18th century, the world around us is full of uncertainties and variations. Suppose we have a pair of integers $N$ and $n$ such that $N \geq n$. Bernoulli commented the expression $B(n; N, p)$ as the probability of $n$ successes with its probability $p$ and $N$-$n$ failures with its probability $q$ by having binomial coefficients $C_N(n)$ which is generally represented with $\binom{N}{n}$. This expression for $N$ trials is in the following form [3, 4]:

$$\begin{aligned} B(n; N, p) &= C_N(n) p^n q^{N-n} \\ &= \binom{N}{n} p^n q^{N-n} \\ &= \binom{N}{n} p^n q^{\acute{n}} \\ &= \frac{N!}{n!\acute{n}!} p^n q^{\acute{n}}, \end{aligned}$$

where $\acute{n} = N - n$, and the relation between the probability of the first event (here spin up), $p$, and the probability of the second event (here spin down), $q$, is $q = 1 - p$.

The concept of uncertainty is present in many complicated problems involving engineering, physics, mathematics, economics, social science and medical sciences etc. Theory of probability is just one of several effective mathematical methods which has been proposed to deal with the problems including uncertainty and imprecise information. In Section 2 we apply this theory for the system of 3 non-interacting spins.
In the past half-century, divers methods in the format of different theories with the names: fuzzy set theory, theory of fuzzy soft sets, rough set theory, theory of vague sets, etc [5, 6, 7, 8, 9, 10, 11] had been introduced which can handle various types of uncertainties. However, Molodtsov [12] pointed out that these methods have their own difficulties

in handling uncertainties, and introduced the appropriate and meaningful concept titled "soft set" which includes technique of parametrization for considering uncertainty problems. In Section 3 we apply soft set theory to the system of 3 non-interacting spins.

The concept of "probabilistic soft set" [13, 14, 15, 16] has been expressed by incorporating both soft set theory and theory of probability so that the set-value mapping in Molodtsov's soft set gives its place to a mapping which have probability distribution as its codomain. The theory of probabilistic soft sets in more details have been studied by [13, 14]. They give some operations such as union, intersections, difference, symmetric difference and Demorgan's laws which hold in this theory respect to these new definitions [13, 14, 15, 16].
In section 4 we utilize the notion of probabilistic soft set to describe the system of 3 non-interacting spins. In many mathematical problems, first we define the new object such as matrix, tensor, set etc. and then it is often useful to compare and note similarity of two objects with the same type like two sets. In this work our aim is to note the study of two spin systems. The mind of several researchers was focused on the problem of similarity measure between fuzzy sets, vague sets and soft sets [17, 18, 19]. In this work our idea is that we visualize the system of non-interacting spins as soft set or probabilistic soft set.

In this paper we study a system consist of three non-interacting spins such as 3 electrons which can be similar with 3 coins. For each coin there exist two possible outcomes or two sides: heads and tails while for each spin we have two situation: up and down. In the following sections we utilize probability theory, soft set theory and probabilistic soft sets, respectively to describe the three spins system in their mathematical language.

# 1 Non-interacting Spins and Probability Theory

The simplest non-trivial system, $S$, that can be investigated by using probability theory is one for which there are only two possible outcomes. Let us label these outcomes with 1 and 2, then we denote their probabilities by $p$ and $q$. The normalization condition gives us

$$p + q = 1. \tag{1}$$

The simplest and well-known physical system with two states is a spin 1/2 (electron). There are two outcomes namely "up" and "down" ($\uparrow$ and $\downarrow$) with equal probabilities 1/2. So, $p = q = 1/2$ for this system.
In solid state physics para and ferromagnet materials are often modeled as spin systems. If we have a system of $N$ spins, then we have $2^N$ possible configurations. Suppose that the spins do not interact with each other while there exist their interaction with the external field. The effect of this interaction can be specified by the energy of any spin, positive and negative energy, so that we have $E_{interaction} = \pm\mu_B B$, depending on whether direction of spin is parallel ($-\mu_B B$) or antiparallel ($+\mu_B B$) to the external field.

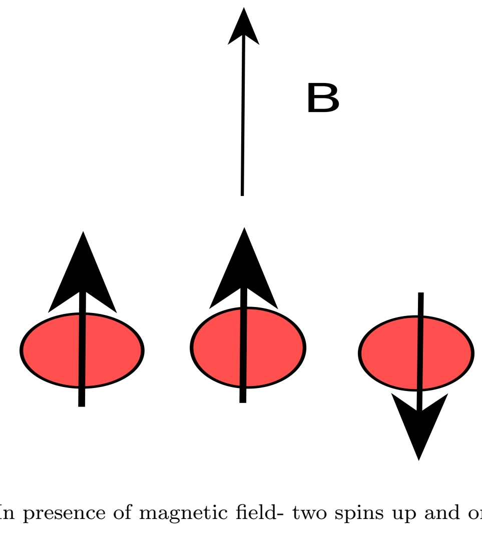


Figure 1: In presence of magnetic field- two spins up and one spin down.

### 1.1 Label energy to the macro-state related to some microstates

The number of non-interacting spins in our system maybe 2,3 or $N$ in general. We should mean two notions: "macro-state" and "microstate". The macro-state of the system is in fact a enumeration of the microstates of it. Note that this model is an isolated system; namely a system in which energy $E$, magnetic field $B$ and number of spins $N$ are fixed. The total number of microstates are enumerated for those that are accessible, it is usually represented by the sign $\Omega = 2^N$. Note that for each particle the orientation of spin can be up ($\uparrow$) or down ($\downarrow$). Two possibilities for each particle give $2^3$ arrangements for $N = 3$. All of the 8 microstates do not belong to just one energy. We should first specify the information related to $E = E_{macrostate}$ and then obtain the accessible microstates. For example, $E_{macrostate} = -\mu B$ is the macroscopic condition for the microstate shown in Figure 1. In this situation (see Figure 1), three of the eight total microstates are accessible as it is shown in Table 1.

### 1.2 Calculate two indicators

The first indicator in the topic of statistics and probability is "mean value" while "standard deviation" as the second indicator gives more information about the system. A spin $S = \frac{1}{2}$ (for electron) has two possible orientations. In the presence of a magnetic field $B$, two states can be labeled with two different energies, namely $E_\uparrow = -\mu_0 B$, and $E_\downarrow = \mu_0 B$. This happening can be explained by the notion of magnetic moment $\mu_0$ [3]. The sign $\uparrow$ means that the moment is parallel to the field ($\overrightarrow{B} = c\overrightarrow{\mu}; c > 0$) and the sign $\downarrow$ means antiparallel ($\overrightarrow{B} = -c\overrightarrow{\mu}; c > 0$).

**Table 1.** Binomial coefficients for each of possible total energy (non-interacting three spins system).

| 1 | 2 | 3 | f | $\acute{f}$ | $C_N(f)$ | $E = -\overrightarrow{\mu}.\overrightarrow{B}$ | (direction of $\overrightarrow{\mu}$), $\vert\mu\vert$ |
|---|---|---|---|---|---|---|---|
| $\uparrow$ | $\uparrow$ | $\uparrow$ | 3 | 0 | 1 | $-3\mu_0 B$ | parallel to $\overrightarrow{B}, 3\mu_0$ |
| $\uparrow$ | $\uparrow$ | $\downarrow$ | 2 | 1 | | $-\mu_0 B$ | |
| $\uparrow$ | $\downarrow$ | $\uparrow$ | 2 | 1 | 3 | $-\mu_0 B$ | parallel to $\overrightarrow{B}, \mu_0$ |
| $\downarrow$ | $\uparrow$ | $\uparrow$ | 2 | 1 | | $-\mu_0 B$ | |
| $\uparrow$ | $\downarrow$ | $\downarrow$ | 1 | 2 | | $+\mu_0 B$ | |
| $\downarrow$ | $\uparrow$ | $\downarrow$ | 1 | 2 | 3 | $+\mu_0 B$ | antiparallel to $\overrightarrow{B}, \mu_0$ |
| $\downarrow$ | $\downarrow$ | $\uparrow$ | 1 | 2 | | $+\mu_0 B$ | |
| $\downarrow$ | $\downarrow$ | $\downarrow$ | 0 | 3 | 1 | $+3\mu_0 B$ | antiparallel to $\overrightarrow{B}, 3\mu_0$ |

$$P(f) = \frac{1}{8}, \frac{3}{8}, \frac{3}{8}, \frac{1}{8}$$

$f$ refers to frequency of "up" spins. $\overline{f} = \sum_{f=0}^{3} P(f)f = (\frac{1}{8} \times 0) + (\frac{3}{8} \times 1) + (\frac{3}{8} \times 2) + (\frac{1}{8} \times 3) = 1.5$

$\overline{f'} = \overline{f} = 1.5$ $\acute{f}$ refers to frequency of "down" spins.

$\overline{(\Delta f)^2} = \sum_{f=0}^{3} P(f)(f - 1.5)^2 = [\frac{1}{8} \times (-1.5)^2] + [\frac{3}{8} \times (1 - 1.5)^2] + [\frac{3}{8} \times (2 - 1.5)^2] + [\frac{1}{8} \times (3 - 1.5)^2] = 0.75$ Thus, by calculating the mean value and standard deviation we may describe a system of non-interacting 3 spins. In the next section we apply the language of soft set for the same system.

## 2 System of Non-interacting Spins and Soft Set

In this section in order to describe a system of non-interacting 3 spins into the language of soft set, we need to use Definition 1 [12, 20, 21], Definition 2 [20, 18], and Lemma 1 [18]. A soft set is not of type classical set, instead it is a pair (a function and a set). [12, 20, 21] Let $U$ and $E$ be an universe set and a set of parameters, respectively. Let $P(U)$ denote the power set of $U$. A pair $S = (F, A)$ is called a soft set over $U$ if and only if $F$ is a mapping of $E$ into the set of all subsets of the set $U$, where $A \subset E$ and $F : A \to P(U)$ is a set-valued mapping. For non-interacting three spins system we denote an universe $U$ and set of parameters $E$ as follows.

$$U = \{\uparrow\uparrow\uparrow, \uparrow\uparrow\downarrow, \uparrow\downarrow\uparrow, \downarrow\uparrow\uparrow, \uparrow\downarrow\downarrow, \downarrow\uparrow\downarrow, \downarrow\downarrow\uparrow, \downarrow\downarrow\downarrow\} \quad (2)$$

$$E = \{three\ spins\ \text{“}up\text{”}, two\ spins\ \text{“}up\text{”}, one\ spin\ \text{“}up\text{”}, no\ spin\ \text{“}up\text{”}\} \tag{3}$$

$e_1 = three\ spins$ "up", $e_2 = two\ spins$ "up", $e_3 = one\ spin$ "up", $e_4 = no\ spin$ "up".
As two sides of a coin in throwing coin are heads or tails, the directions of each spin in the presence of magnetic field can be up or down. We have the equivalent relations:

- Three spins up ≡ no spin down
- Two spins up ≡ one spin down
- One spin up ≡ two spins down
- No spin up ≡ three spins down

For non-interacting three spins system we have $F(e_1) = F(three\ spins\ \text{“}up\text{”}) = \{\uparrow\uparrow\uparrow\}$ $F(e_2) = F(two\ spins\ \text{“}up\text{”}) = \{\uparrow\uparrow\downarrow, \uparrow\downarrow\uparrow, \downarrow\uparrow\uparrow\}$ $F(e_3) = F(one\ spin\ \text{“}up\text{”}) = \{\uparrow\downarrow\downarrow, \downarrow\uparrow\downarrow, \downarrow\downarrow\uparrow\}$ $F(e_4) = F(no\ spin\ \text{“}up\text{”}) = \{\downarrow\downarrow\downarrow\}$ To compare two spin systems we use the definition of similarity measure $S_m$. [18, 20] Let $(S1, E)$ and $(S2, E)$ be two soft spin sets. Then the similarity measure between $S1$ and $S2$ is denoted by $S_m(S1, S2)$ and

$$S_m(S1, S2) = \frac{\sum_i \overrightarrow{S1}(c_i) \bullet \overrightarrow{S2}(c_i)}{\sum_i [\overrightarrow{S1}(c_i)^2 \vee \overrightarrow{S2}(c_i)^2]} \tag{4}$$

[18, 21] Let $(S1, E)$ and $(S2, E)$ be two soft sets over the same finite universe $U$. Then the following hold:

1. $S_m(S1, S2) = S_m(S2, S1)$
2. $0 \leq S_m(S1, S2) \leq 1$
3. $S_m(S1, S1) = S_m(S2, S2) = 1$

Let us consider each of spin systems as a soft spins set, then a three spins system may be represented by a three components column vector for each spin. The number 1 means up spin and number 0 means down spin. Similarity measure is a convenient indicator for comparing similarity between two spins systems. Similarity measures greater that 0.5 are acceptable as good indicator. To clarify our idea we give three examples in which we use initial universe set $U$ given in Eq. (3.1). Calculate similarity measure for two soft spins sets $(S1, E)$ and $(S2, E)$ over $U$, where $S1 = S2$ and each three-component column represents orientation of three spins in which 1 means up and 0 means down, namely: $\uparrow\uparrow\downarrow \equiv \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ Let $S1 = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$ and $S2 = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$ be their representing matrices and $E = \{e_1, e_2\}$,

$e_1 = \uparrow\uparrow\uparrow \equiv \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, $e_2 = \downarrow\uparrow\uparrow \equiv \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$

$$S_m(S1, S2) = \frac{\sum_{i=1}^{3} \overrightarrow{S1}(c_i) \bullet \overrightarrow{S2}(c_i)}{\sum_{i=1}^{3} [\overrightarrow{S1}(c_i)^2 \vee \overrightarrow{S2}(c_i)^2]} = \frac{3+2+2}{3+2+2} = 1$$

where $c_1, c_2, c_3$ mean the first, second and third column in matrix, respectively.

Having two soft spin sets $(S1, E)$ and $(S2, E)$ over $U$, calculate their similarity measure.

Let $S1 = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ and $S2 = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ be their representing matrices and $E = \{e_1, e_2\}$

$S_m(S1, S2) = \frac{\sum_{i=1}^{3} \overrightarrow{S1}(c_i) \bullet \overrightarrow{S1}(c_i)}{\sum_{i=1}^{3} [\overrightarrow{S1}(c_i)^2 \vee \overrightarrow{S2}(c_i)^2]} = \frac{3+2+2}{3+2+3} = \frac{7}{8} = 0.875$ Having two soft spin sets $(S1, E)$ and $(S2, E)$ over $U$, calculate their similarity measure

Let $S1 = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$ and $S2 = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$ be their representing matrices and $E = \{e_1, e_2\}$

$$\mathrm{S}_m(S1, S2) = \frac{\sum_{i=1}^{4} \overrightarrow{S1}(c_i) \bullet \overrightarrow{S2}(c_i)}{\sum_{i=1}^{4} [\overrightarrow{S1}(c_i)^2 \vee \overrightarrow{S2}(c_i)^2]} = \frac{3+2+2+2}{3+3+3+2} = \frac{9}{11} = 0.818$$

## 3 System of Non-interacting Spins and Probabilistic Soft Set Theory

In this section we need to use the following definition from [13, 14]. We can describe 3 spins system by the notion of probabilistic soft set. [13, 14] Let $U$ be an universe set and $E$ a set of parameters with respect to $U$. A probabilistic soft set over $U$ is defined as a pair $(F, A)$ consisting of a subset $A$ of $E$ and a function $F$ representing a $F : E \to Pr(U)$. Hence probabilistic soft set F over U can be represented by the set of ordered pairs F={(x, f): x$\in E, f \in Pr(U)$} Using Definition 3, we can form the following table. Representing system of spins by probabilistic soft set give us opportunity for better understanding of a system as it is shown in Table 2.

**Table 2.** Description of non-interacting three spins system by probabilistic soft set.

| $U$ | ↑↑↑ | ↑↑↓ | ↑↓↑ | ↓↑↑ | ↑↓↓ | ↓↑↓ | ↓↓↑ | ↓↓↓ |
|---|---|---|---|---|---|---|---|---|
| 3 spins up | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 spins up | 0 | $0.\bar{3}$ | $0.\bar{3}$ | $0.\bar{3}$ | 0 | 0 | 0 | 0 |
| 1 spins up | 0 | 0 | 0 | 0 | $0.\bar{3}$ | $0.\bar{3}$ | $0.\bar{3}$ | 0 |
| 0 spin up | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |

F(3 spins up)=$1_{\overline{\uparrow\uparrow\uparrow}}$ F(2 spins up)=$0.\bar{3}_{\overline{\uparrow\uparrow\downarrow + \frac{0.\bar{3}}{\uparrow\downarrow\uparrow} + \frac{0.\bar{3}}{\downarrow\uparrow\uparrow}}}$ F(1 spin up)=$0.\bar{3}_{\overline{\downarrow\downarrow\uparrow + \frac{0.\bar{3}}{\downarrow\uparrow\downarrow} + \frac{0.\bar{3}}{\uparrow\downarrow\downarrow}}}$ F(0 spin up)=$1_{\overline{\downarrow\downarrow\downarrow}}$ Thus Table 2 gives classified information which describes our spin system into the language of probabilistic soft set.

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