Nonlinear Vibration of Piezoelectric Nano Biological Sensor Based on Non-Classical Mathematical Approach

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Abstract

In this study, nonlinear vibration analysis of a parametrically excited piezoelectric nano beam subjected to DC and AC voltages is investigated for biological sensor applications on the basis of the non-local continuum theory. Equations of the motion and boundary conditions of the nano beam are obtained by implementation of Hamilton's principle and the Galerkin approach. Hamiltonian solution namely Frequency-Amplitude approach is used for natural frequencies and mode shapes as a function of the piezo-layered nano beam characteristic non-local size scale parameter. The size effects on the vibration behavior (frequency and harmonic response) of the beam are studied and it is found that the non-local parameter has significant effects on the free vibration of system.

Keyword: Piezoelectric Nano biological sensor, Nonlocal continuum theory, Size scale parameter, vibration response, Hamiltonian, Frequency-Amplitude approach.

1. Introduction

In recent years, piezoelectric nano/micro-structures such as nano-beams, nano-membranes and nano-shells have been fabricated, and are attracting worldwide attention in nano/micro-electromechanical (NEMS/MEMS) systems [1]. Analysis of nonlinear dynamics and vibration of NEMS/MEMS is one of the important issues in such systems which unlike excitation forces of the external excitation, the force here appear as time-varying coefficients or parameters in the differential equation. In the past decade, investigating the nonlinear dynamics and vibration on the nano/micro-structures has become one of the attractive research areas in nano/micro-mechanics.

In this case, a concept of mass sensing based on amplification of parametric resonance is proposed by Turner and Zhang [2]. Also, the dynamic responses and behavior of parametric resonance of a micro- oscillator is investigated by Zhang et al. [3]. Dynamics of an electrostatically actuated MEMS resonant sensor under parametric and external excitations is presented by Zhang et al. using the method of multiple scales [4]. The deflection and natural frequency of a new piezoelectric sensor–actuator system combined electrostatic and piezoelectric is investigated by Zamanian *et al.* using the Hamilton principle and the Galerkin method [5]. Also, Rezaei Kivi et al. presented electrostatic and piezoelectric excitations and static and dynamic pull-in instability on a micro-gripper and an FGP micro-beam, respectively [6]. Pure parametric excitation of the micro cantilever beam actuated by piezoelectric layer is studied by Ghazavi et al. [7]. Non-Fourier heat conduction model based on continuum theory is used for by Saeedi Vahdat et al. to study thermoelastic damping (TED) in a micro-beam resonator applying Galerkin and complex-frequency approaches [8]. Also, Azizi et al. investigated nonlinear dynamics of a parametrically excited piezoelectric mass-sensor which discretized to a nonlinear damped Mathieu equation and were excited by a combination of a DC and an AC voltage [9].

Moreover, experiments at the nano/microscale are extremely difficult to conduct and atomistic dynamic modeling is strongly restricted by computational capacities. Therefore, high-order continuum theories have emerged as an applicable approach to analyze the dynamic behavior of nano/microsystems, which have attracted much interest in recent years. Jiang and Yan [10] employed the surface elasticity theory to study the static behavior of nanowires. They presented explicit solutions to analyses surface effects on the deflection of the beam. Moreover, Eringen [11, 12] proposed non-local continuum mechanics formulation which has been extensively used to investigate the size effect on small-scale systems [13-15]. Nonlinearity in NEMS/MEMS may lead to major problems in calculating and analyzing the governing equations, especially strongly nonlinear system. Recently, some approximate methods are considered to be the powerful methods capable of handling strongly nonlinear behaviors, especially in NEMS/MEMS systems and can converge to an accurate periodic solution for smooth nonlinear systems that can be showed in works of Hashemi Kachapi et al. in references [16-19].

In this study, nonlinear vibration analysis of a parametrically excited piezoelectric nano beam subjected to DC and AC voltages is investigated on the basis of the non-local continuum theory and using Hamilton's and Galerkin principle and also Hamiltonian solution namely Frequency-Amplitude approach is used for analysis of nonlinear vibration.

3. Nonlocal model of piezoelectric nano beam

A piezoelectric nano beam with geometric and material property, i.e., length l, width a, thickness h, density ρ Young's modulus E, thickness of two piezoelectric layers h_p, density ρ_p is shown in Fig. 1. Also, E_p, e_p, ξ_p, V_{DC} and V_{AC} are Young's modulus, the piezoelectric constant, the dielectric constant, DC and AC voltages of piezoelectric layers, respectively. The coordinate system is attached to the middle of the left end of the piezo-nano beam where *x* and *z* refer to a horizontal and vertical coordinate, respectively.as illustrated in Fig. 1,

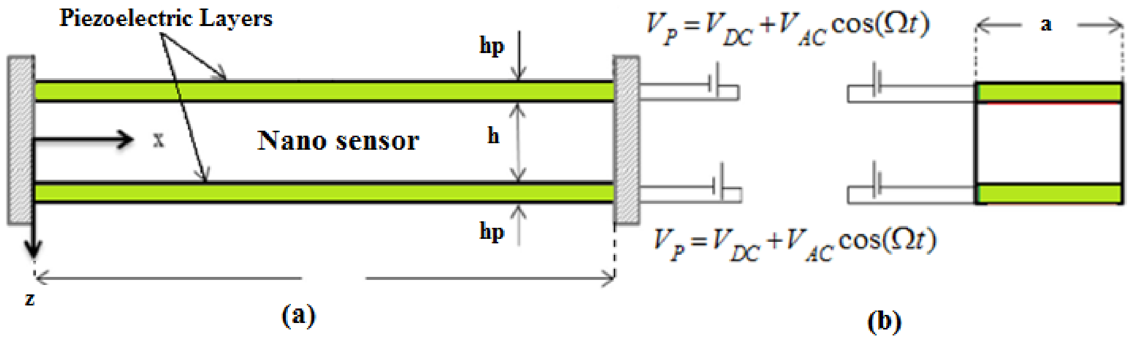

Fig. 1. piezoelectric nano sensor: (a) Front view (b) Side view

The total potential strain energy of the nano-beam can be expressed as [9]:

$$U = U_b + U_a + U_P \tag{1}$$

Where U_b, U_P and U_a are potential energy due to the bending, piezoelectric actuation axial force and the mid-plane stretching axial force, respectively, and expressed as:

$$U_b = \frac{(EI)_{eq}}{2}\int_{x=0}^{x=l}\left(\frac{\partial^2 w}{\partial x^2}\right)^2 dx \tag{2}$$

$$U_a = \frac{(EA)_{eq}(l'-l)}{4l}\int_0^l\left(\frac{\partial w}{\partial x}\right)^2 dx \tag{3}$$

$$U_P = -\frac{2V_P a(l'-l)}{h}\int_0^{h_P} e_p dh \tag{4}$$

Where

$$(EI)_{eq} = EI + E_P hah_P\left(\frac{h}{2}+h_P\right) \tag{5a}$$

$$(EA)_{eq} = Eah + 2E_P ah_P \tag{5b}$$

Also, the kinetic energy of the piezo-nano beam is introduced as:

$$T = \frac{(\rho ah)_{eq}}{2}\int_0^l\left(\frac{\partial w}{\partial t}\right)dx \tag{6}$$

where $(\rho ah)_{eq} = \rho ah + 2\rho_P ah_P$.

The work $W_{n.c.}$ due to the external damping can be presented as:

$$W_{n.c.}=\frac{1}{2}\int_0^l(-f_{vis}w)dx \tag{7}$$

where $f_{vis}=\mathrm{B}(\partial w/\partial t)$ is considered as the damping force and B is the viscous damping constant of the system [7].

The total partial differential equation of motion is obtained by the Hamiltonian as:

$$\int_0^t \delta(\mathrm{T}-\mathrm{U}+\mathrm{W}_{n.c.})dt=0 \tag{8}$$

$$\int_0^t\left\{\begin{array}{l}-(EI)_{eq}w''\delta w'\big|_0^l+(EI)_{eq}w'''\delta w\big|_0^l-(EI)_{eq}\int_0^l w^{IV}\delta w dx\\ -F_P w'\delta w\big|_0^l+F_P w''\delta w dx-\dfrac{(cA)_{eq}}{2}\int_0^l w'^2 dx w'\delta w\big|_0^l\\ +\dfrac{(EA)_{eq}}{2}\int_0^l w'^2 dx\int_0^l w''\delta w dx\end{array}\right\}dt=0 \tag{9}$$

$$\int_0^l\left\{(\rho A)_{eq}\dot{w}\delta w\big|_0^t-(\rho A)_{eq}\int_0^t\ddot{w}\delta w dt\right\}dx$$

Including the effect of viscous damping, the governing equation and the corresponding boundary conditions reduce to:

$$\begin{aligned}&(EI)_{eq}\frac{\partial^4 w(x,t)}{\partial x^4}+(\rho A)_{eq}\frac{\partial^2 w(x,t)}{\partial t^2}\\&-\left(F_P+\frac{(EA)_{eq}}{2l}\int_0^l\left(\frac{\partial w}{\partial x}\right)^2dx\right)\frac{\partial^2 w(x,t)}{\partial x^2}+\mathrm{B}\frac{\partial w(x,t)}{\partial t}=0\end{aligned} \tag{10}$$

$$w(0,t)=w(l,t)=0,\quad \frac{\partial w(o,t)}{\partial t}=\frac{\partial w(l,t)}{\partial t}=0 \tag{11}$$

Equation (10) rewrite and resulted in;

$$(\rho A)_{eq}\frac{\partial^2 w(x,t)}{\partial t^2}-\left(F_P+\frac{(EA)_{eq}}{2l}\int_0^l\left(\frac{\partial w}{\partial x}\right)^2dx\right)\frac{\partial^2 w(x,t)}{\partial x^2}+\mathrm{B}\frac{\partial w(x,t)}{\partial t}=\frac{\partial^2 M_{eq}}{\partial x^2} \tag{12}$$

$$M_{eq}\delta\left(\frac{\partial w(x,t)}{\partial t}\right)\Big|_0^l=0,\qquad \frac{\partial M_{eq}}{\partial x}\delta w(x,t)\Big|_0^l=0 \tag{13}$$

Where $M_{eq} = M_b + M_p = -(EI)_{eq} \dfrac{\partial^2 w(x,t)}{\partial x^2}$ is the effective moment of the system and are considered as?

$$M_{x,b} = \iint_{A_b} \sigma_{xx,b} z \mathrm{d}A_b \qquad (14)$$
$$M_{x,\mathrm{p}} = \iint_{A_p} \sigma_{xx,p} z \mathrm{d}A_p$$

For the beam with the piezo layers, the non-local constitutive relationships can be stated in a one-dimensional (1D) form as [11-12]:

$$\sigma_{xx,b} - \mu^2 \frac{\partial^2 \sigma_{xx,b}}{\partial x^2} = E\,\varepsilon_{xx} \qquad (15)$$

$$\sigma_{xx,\mathrm{p}} - \mu^2 \frac{\partial^2 \sigma_{xx,\mathrm{p}}}{\partial x^2} = E_p \varepsilon_{xx} - e_p E_z \qquad (16)$$

$$D_z - \mu^2 \frac{\partial^2 D_z}{\partial x^2} = e_p \varepsilon_{xx} + \xi_p E_z \qquad (17)$$

Where μ is nonlocal parameter. By integrating (15)– (16) with (14) and combining them, we can obtain

$$M_{eq} - \mu^2 \frac{\partial^2 M_{eq}}{\partial x^2} = -(EI)_{eq} \frac{\partial^2 w}{\partial x^2} - e_p Q_p E_z \qquad (18)$$

where the following relationships are considered

$$\left(I_b, I_p\right) = \iint_{\left(A_b, A_p\right)} z^2 \left(\mathrm{d}A_b, \mathrm{d}A_p\right) \qquad (19)$$
$$Q_p = \iint_{A_p} z \mathrm{d}A_p$$

Moreover, submitting (18) into (12) gives

$$M_{eq} = -(EI)_{eq} \frac{\partial^2 w}{\partial x^2} - e_p Q_p E_z \qquad (20)$$
$$+\mu^2 \left[(\rho A)_{eq} \frac{\partial^2 w(x,t)}{\partial t^2} - \left(F_P + \frac{(EA)_{eq}}{2l} \int_0^l \left(\frac{\partial w}{\partial x} \right)^2 dx \right) \frac{\partial^2 w(x,t)}{\partial x^2} + \mathrm{B} \frac{\partial w(x,t)}{\partial t} \right]$$

Also, by substituting (20) into (12) and (13), the equation of the motion and boundary conditions of the piezo-layered nano beam can be concluded as

$$T\left(w(x,t)\right)=(\rho A)_{eq}\left(1-\mu^2\frac{\partial^2}{\partial x^2}\right)\frac{\partial^2 w(x,t)}{\partial t^2}+(EI)_{eq}\frac{\partial^4 w}{\partial x^4}+\mathrm{B}\left(1-\mu^2\frac{\partial^2}{\partial x^2}\right)\frac{\partial w(x,t)}{\partial t}$$
$$-\left(F_P+\frac{(EA)_{eq}}{2l}\int_0^l\left(\frac{\partial w}{\partial x}\right)^2dx\right)\left(1-\mu^2\frac{\partial^2}{\partial x^2}\right)\frac{\partial^2 w(x,t)}{\partial x^2}-K_p\frac{\partial^2 V_p}{\partial x^2}=0 \tag{21}$$

$$\left[-(EI)_{eq}\frac{\partial^4 w}{\partial x^4}+\mu^2(\rho A)_{eq}\frac{\partial^2 w(x,t)}{\partial t^2}\right]\delta\left(\frac{\partial w(x,t)}{\partial x}\right)\Big|_0^L=0 \tag{22}$$
$$\frac{\partial}{\partial x}\left[-(EI)_{eq}\frac{\partial^4 w}{\partial x^4}+\mu^2(\rho A)_{eq}\frac{\partial^2 w(x,t)}{\partial t^2}\right]\delta w(x,t)\Big|_0^L=0$$

where V_p is the piezoelectric applied voltage and $K_p=-e_pQ_p/h_p$ is a constant coefficient? There deflection is no exact solution to Eq. (22). To approximate the homoclinic trajectory of Eq. (22), Galerkin method is used to discretize this equation; therefore, the approximate solution is supposed to be in the form

$$w(x,t)=\sum_{i=1}^{n}\phi_i(x)u_i(t) \tag{23}$$

The deflection $w(x,t)$ in Eq. (23) is expressed as a sum of spatial shapes that, a priori, satisfy the imposed kinematic boundary conditions and n is the number of degrees of freedom, $\phi_i(x)$ is the i eigenfunction of the beam and $u_i(\tau)$ is the i time dependent deflection parameter of the beam. Based on a single degree-of-freedom model of the beams ($\mathrm{n}=1$), Eq. (23) can be solved with appropriate accuracy.

For example, $\phi(x)$ can be assumed as

$$\phi(x)=16x^2(1-x)^2 \tag{24}$$

or

$$\phi(x)=\left(\cosh\left(\frac{\alpha x}{l}\right)-\cos\left(\frac{\alpha x}{l}\right)\right)-\left(\frac{\cosh\alpha-\cos\alpha}{\sinh\alpha-\sin\alpha}\right)\left(\sinh\left(\frac{\alpha x}{l}\right)-\sin\left(\frac{\alpha x}{l}\right)\right),\quad \alpha=4.730040745 \tag{25}$$

Eqs. (24) or (25) are the first eigenfunction of a double-clamped beam that satisfy all the kinematic boundary conditions. The one-parameter Galerkin method can be computed by:

$$\int_0^1\phi(x)T\left(w(x,t)\right)dx=0 \tag{26}$$

After substituting for $T\left(w(x,t)\right)$ from Eq. (21) into Eq. (26), multiplying by $\phi(x)$, and integrating the outcome from 0 to 1, we obtain

$$\int_0^1 \phi(x)\begin{pmatrix} \left(\rho A\phi(x)-\mu^2\rho A\phi''(x)\right)\ddot{u}(t)+\left(B\phi(x)-\mu^2 B\phi''(x)\right)\dot{u}(t)+ \\ \left(EI\phi''''(x)-F_p\,\phi''(x)+F_p\,\mu^2\phi''''(x)\right)u(t)+\begin{pmatrix} -\dfrac{EA\phi''(x)}{2l}\left(\displaystyle\int_0^1 \phi'^2(x)dx\right) \\ +\dfrac{EA\mu^2\phi''''(x)}{2l}\left(\displaystyle\int_0^1 \phi'^2(x)dx\right)\end{pmatrix}u(t)^3 \\ -K_pV_p''(x,t)\end{pmatrix}dx=0 \tag{27}$$

Keeping the first mode in the modal expansion to Eq. (27), yields to nonlinear equation as follows:

$$\begin{aligned}&\left(\alpha_1-\mu^2\alpha_2\right)\ddot{u}(t)+\left(\alpha_3-\mu^2\alpha_4\right)\dot{u}(t)+\left(\alpha_5+\alpha_6-\mu^2\alpha_7\right)u(t)\\ &+\left(\alpha_8-\mu^2\alpha_9\right)u(t)^3=K_p\frac{\partial^2 V_p(x,t)}{\partial x^2}\end{aligned} \tag{28}$$

where:

$$\begin{aligned}&\alpha_1=\int_0^1\phi^2\rho A\,dx,\quad \alpha_2=\int_0^1\rho A\,\phi\phi''dx,\quad \alpha_3=\int_0^1 B\phi^2dx,\quad \alpha_4=\int_0^1 B\phi\phi''dx,\\ &\alpha_5=\int_0^1 EI\,\phi\phi^{IV}dx,\quad \alpha_6=-\int_0^1 F_p\,\phi\phi''dx,\quad \alpha_7=-\int_0^1 F_p\,\phi\phi^{IV}dx,\\ &\alpha_8=-\int_0^1\frac{EA\phi\phi''}{2l}\left(\int_0^1\phi'^2dx\right)dx,\qquad \alpha_9=-\int_0^1\frac{EA\phi\phi^{IV}}{2l}\left(\int_0^1\phi'^2dx\right)dx\end{aligned} \tag{29}$$

where Eq. (28) present completely the flexural vibration equation for nano beam with piezo-layered actuators for mass sensing applications.

4. Solution Approach

In this Section, the size-dependent nonlinear vibration of a nano beam is simulated. The geometry and material properties of the beam and the piezo actuator are listed in Table 1.

Table 1. Material and geometric properties of nano beam and piezoelectric actuator

Geometrical and mechanical property	nano beam (SiO_2)	Piezoelectric layers (PZT)

Density, (ρ)	$2330\ kg\ /\ m^3$	$7500\ kg\ /\ m^3$
Young`s modulus (E)	$107\ GPa$	$139\ GPa$
Piezoelectric constant (e_p)	-----	$123\times10^{-12}\ m\ /V$
Height (h)	$60\ nm$	$20\ nm$
Length (L)	$300\ nm$	$300\ nm$
Width (a)	$60\ nm$	$60\ nm$

To solve the free vibration problem and study the size-dependent behavior of the system, a harmonic solution for the damped transverse displacement Eq. (28) is the first order periodic solution in the form of:

$$u(t) = A\cos\omega t \tag{30}$$

Which A is oscillator amplitude and ω is natural frequency of piezo-actuated non-local Nano beam which will be obtained.

And the initial conditions take the form of:

$$u(0) = A\ ,\ \dot{u}(0) = 0 \tag{31}$$

In this section, we use the Frequency-Amplitude Formulation to study size-dependent nonlinear vibration of a Nano beam mentioned in Eq. (28). According to Frequency-Amplitude Formulation, we choose two trial functions $u_1(t) = A\cos(t)$ and $u_2(t) = A\cos(\omega t)$ which the solutions of the following linear equations are, respectively:

$$\ddot{u}(t) + \omega_1^2 u(t) = 0, \qquad \omega_1^2 = 1 \tag{32}$$

$$\ddot{u}(t) + \omega_2^2 u(t) = 0, \qquad \omega_2^2 = \omega^2 \tag{33}$$

The trial functions substituted in main equation (28) that the residuals are:

$$R_1(t) = \begin{pmatrix} A^2\left(\alpha_8 - \mu^2\alpha_9\right)\cos^3(t) + \left(\left(\alpha_2 - \mu^2\alpha_7\right)\mu^2 - \alpha_1 + \alpha_5 + \alpha_6\right)\cos(t) \\ +\left(-\alpha_3 + \mu^2\alpha_4\right)\sin(t) \end{pmatrix} A \tag{34}$$

$$R_2(t)=\begin{pmatrix} A^2\left(\alpha_8-\mu^2\alpha_9\right)\cos^3(\omega t)+\left(\left(-\alpha_1+\mu^2\alpha_2\right)\omega^2+\alpha_5+\alpha_6-\mu^2\alpha_7\right)\cos(\omega t) \\ +\left(-\alpha_3+\mu^2\alpha_4\right)\omega\sin(\omega t) \end{pmatrix}A \tag{35}$$

We use the method of weighted residuals to overcome the shortcoming. To this end, we introduce two new residual variables $\overset{:}{R}_1$ and $\overset{:}{R}_2$ defined as

$$\overset{:}{R}_1=\frac{4}{T_1}\int_0^{T_1/4}R_1(t)\cos\left(\frac{2\pi}{T_1}t\right)dt\ ,\quad T_1=2\pi \tag{36}$$

and

$$\overset{:}{R}_2=\frac{4}{T_2}\int_0^{T_2/4}R_2(t)\cos\left(\frac{2\pi}{T_2}t\right)dt\ ,\quad T_2=\frac{2\pi}{\omega} \tag{37}$$

We can approximately determine ω^2 in the form

$$\omega^2=\frac{\omega_1^{\,2}\,\overset{:}{R}_2-\omega_2^{\,2}\,\overset{:}{R}_1}{\overset{:}{R}_2-\overset{:}{R}_1} \tag{38}$$

Substituted Eqs. (34) and (35) in Eqs. (36) and (37) that can be expressed as:

$$\overset{:}{R}_1(t)=\frac{A}{\pi}\begin{pmatrix} -\alpha_3-\frac{1}{2}\alpha_1\pi+\frac{3}{8}A^2\alpha_8\pi+\frac{1}{2}\alpha_6\pi+\frac{1}{2}\alpha_5\pi \\ +\mu^2\left(\alpha_4+\frac{1}{2}\alpha_2\pi-\frac{1}{2}\alpha_7\pi-\frac{3}{8}A^2\alpha_9\pi\right) \end{pmatrix} \tag{39}$$

and

$$\overset{:}{R}_2(t)=\frac{A}{\pi}\begin{pmatrix} -\omega\alpha_3-\frac{1}{2}\omega^2\alpha_1\pi+\frac{3}{8}A^2\alpha_8\pi+\frac{1}{2}\alpha_6\pi+\frac{1}{2}\alpha_5\pi \\ +\mu^2\left(\omega\alpha_4+\frac{1}{2}\omega^2\alpha_2\pi-\frac{1}{2}\alpha_7\pi-\frac{3}{8}A^2\alpha_9\pi\right) \end{pmatrix} \tag{40}$$

Substituting Eqs (32), (33), (39) and (40) into Eq. (38) leads to:

$$\omega^2=\frac{\begin{pmatrix}-\omega\alpha_3-\frac{1}{2}\omega^2\alpha_1\pi+\frac{3}{8}A^2\alpha_8\pi+\frac{1}{2}\alpha_6\pi+\frac{1}{2}\alpha_5\pi\\+\mu^2\left(\omega\alpha_4+\frac{1}{2}\omega^2\alpha_2\pi-\frac{1}{2}\alpha_7\pi-\frac{3}{8}A^2\alpha_9\pi\right)\end{pmatrix}-\omega^2\begin{pmatrix}-\alpha_3-\frac{1}{2}\alpha_1\pi+\frac{3}{8}A^2\alpha_8\pi+\frac{1}{2}\alpha_6\pi+\frac{1}{2}\alpha_5\pi\\+\mu^2\left(\alpha_4+\frac{1}{2}\alpha_2\pi-\frac{1}{2}\alpha_7\pi-\frac{3}{8}A^2\alpha_9\pi\right)\end{pmatrix}}{\begin{pmatrix}-\omega\alpha_3-\frac{1}{2}\omega^2\alpha_1\pi+\frac{3}{8}A^2\alpha_8\pi+\frac{1}{2}\alpha_6\pi+\frac{1}{2}\alpha_5\pi\\+\mu^2\left(\omega\alpha_4+\frac{1}{2}\omega^2\alpha_2\pi-\frac{1}{2}\alpha_7\pi-\frac{3}{8}A^2\alpha_9\pi\right)\end{pmatrix}-\begin{pmatrix}-\alpha_3-\frac{1}{2}\alpha_1\pi+\frac{3}{8}A^2\alpha_8\pi+\frac{1}{2}\alpha_6\pi+\frac{1}{2}\alpha_5\pi\\+\mu^2\left(\alpha_4+\frac{1}{2}\alpha_2\pi-\frac{1}{2}\alpha_7\pi-\frac{3}{8}A^2\alpha_9\pi\right)\end{pmatrix}} \tag{41}$$

That solution of Eq. (41), its approximate frequency leads to:

$$\omega=\frac{0.3183\left(\alpha_3-\mu^2\alpha_4\right)\begin{pmatrix}39.4784\left(-\mu^2\alpha_2\alpha_5+\mu^4\alpha_2\alpha_7-\mu^2\alpha_2\alpha_6+\alpha_1\alpha_5+\alpha_1\alpha_6-\mu^2\alpha_1\alpha_7\right)\\29.6088A^2\left(-\mu^2\alpha_2\alpha_8+\mu^4\alpha_2\alpha_9+\alpha_1\alpha_8-\mu^2\alpha_1\alpha_9\right)\\+4\mu^4\alpha_4^2-8\mu^2\alpha_4\alpha_3+4\alpha_3^2\end{pmatrix}^{1/2}}{-\alpha_1+\mu^2\alpha_2} \tag{42}$$

Where ω will be used in Eq. (30) as the first order of harmonic solution.

5. Simulations and results

In this section the result which obtained using the Frequency-Amplitude approach is investigated. In following, the effect of design parameters on the nonlinear frequency and steady state response or period of motion of nonlinear vibration in the nano beam with piezo-layered actuator for mass sensing applications, such as $\alpha_i\ (i=1\ldots 9)$ to investigated and show in several cases.

In Figs. 2-4, the effects of nonlocal parameter μ on the natural frequency and free vibration response of nano beam for $\alpha_i=1(i=1\ldots 9)$, $A=1$ and $t=1$ are investigated and in Figs 5-16 , the effects of $\alpha_i\ (i=1\ldots 9)$ parameters on the natural frequency and free vibration response of nano beam for local case and different values on nonlocal parameter μ are investigated. According to Fig. 2, it can be seen that the frequency response of the piezo-actuated nano beam decrease with increase of nonlocal parameter μ .

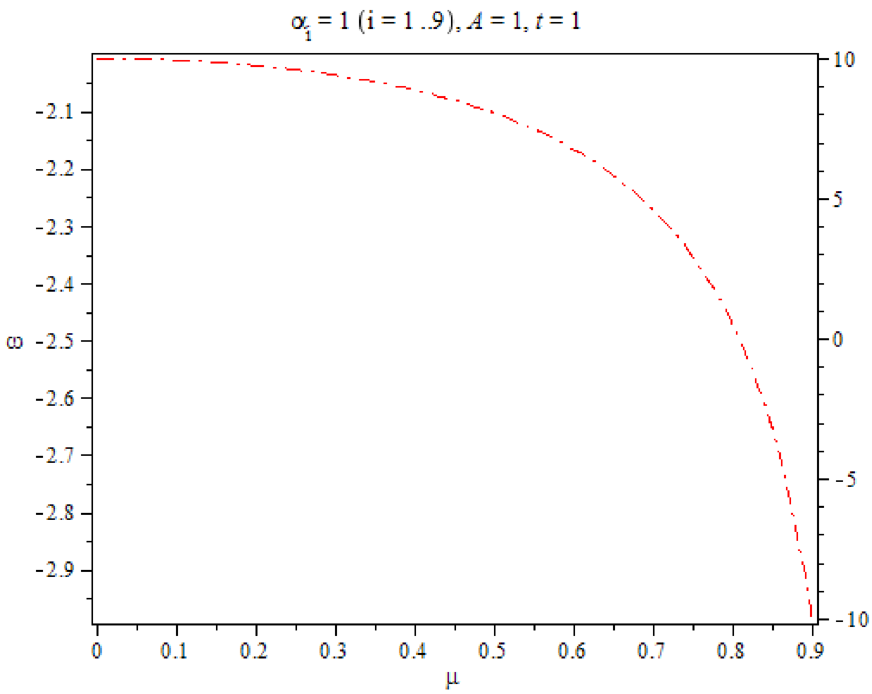

Fig.2. The effects of nonlocal parameter μ on the natural frequency of nano beam for $\alpha_i = 1(i = 1 \ldots 9)$, $A = 1$ and $t = 1$

In Fig. 3 as shown, for values $\mu < 1$, as the frequency, the harmonic response of system decreases and in the vicinity of $\mu = 1$ a sudden leap happens and then with less steep decreases.

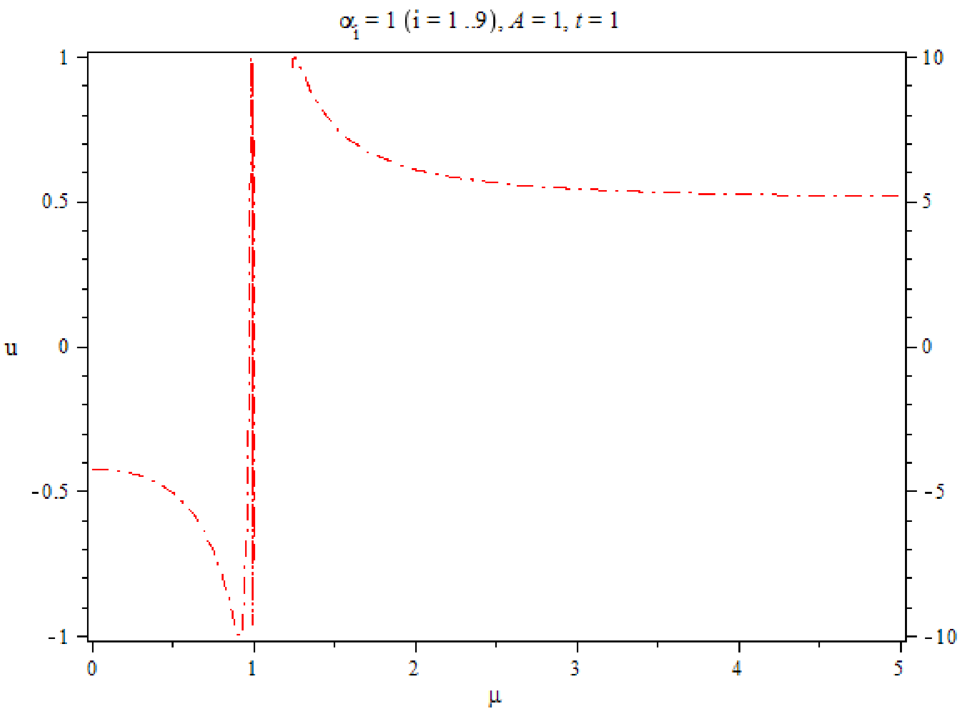

Fig.3. The effects of nonlocal parameter μ on the free vibration response of nano beam for $\alpha_i = 1(i = 1L\ 9)$, $A = 1$ and $t = 1$

Fig. 4 show that by increasing in values of nonlocal parameter μ, the number of oscillations of the harmonic response of the piezo-actuated nano increase.

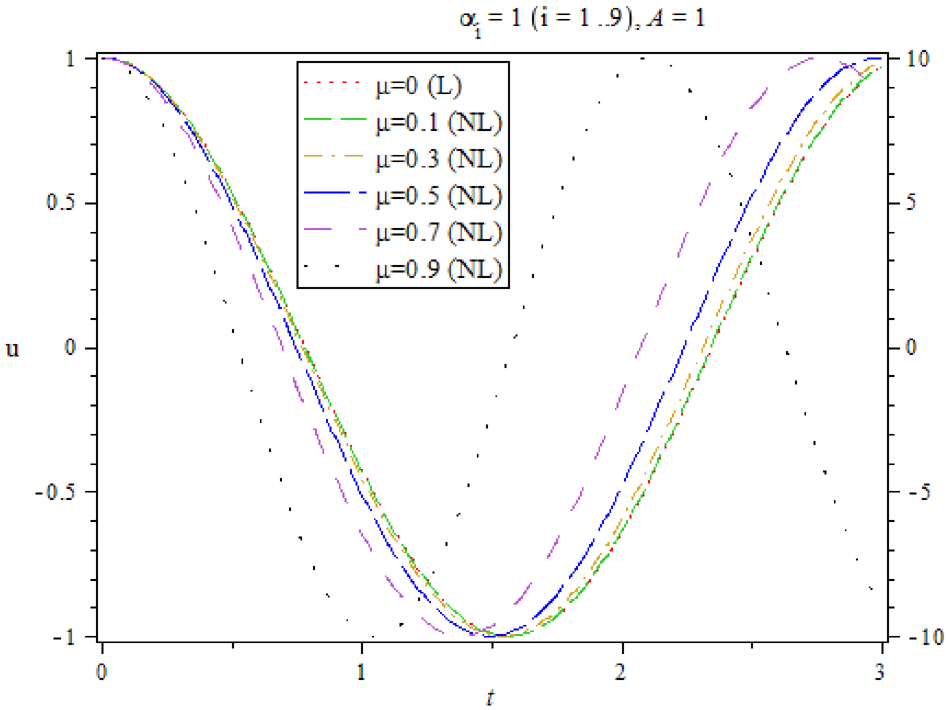

Fig.4. The free vibration response of nano beam for different values of nonlocal parameter μ and $\alpha_i = 1(i = 1L\ 9)$, $A = 1$ and $t = 1$

Figs. 5-8 show the effects of α_1 and α_2 parameters on the frequency and the free vibration response of nano beam for constant values of other parameters. According to Eq. (29), parameters of α_1 and α_2 dependent on variation in ρ and A.

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Fig.5. The effects of nonlocal parameter μ on the free vibration response of nano beam for $\alpha_i = 1(i = 1L\ 9)$, $A = 1$ and $t = 1$

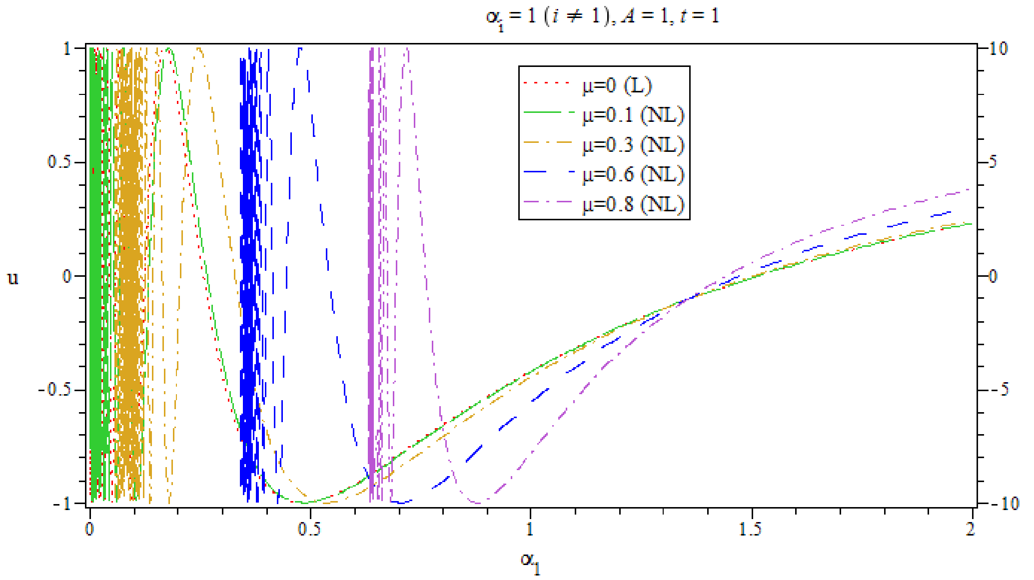

Fig.6. The effects of nonlocal parameter μ on the free vibration response of nano beam for $\alpha_i = 1 (i = 1 \ldots 9)$, $A = 1$ and $t = 1$

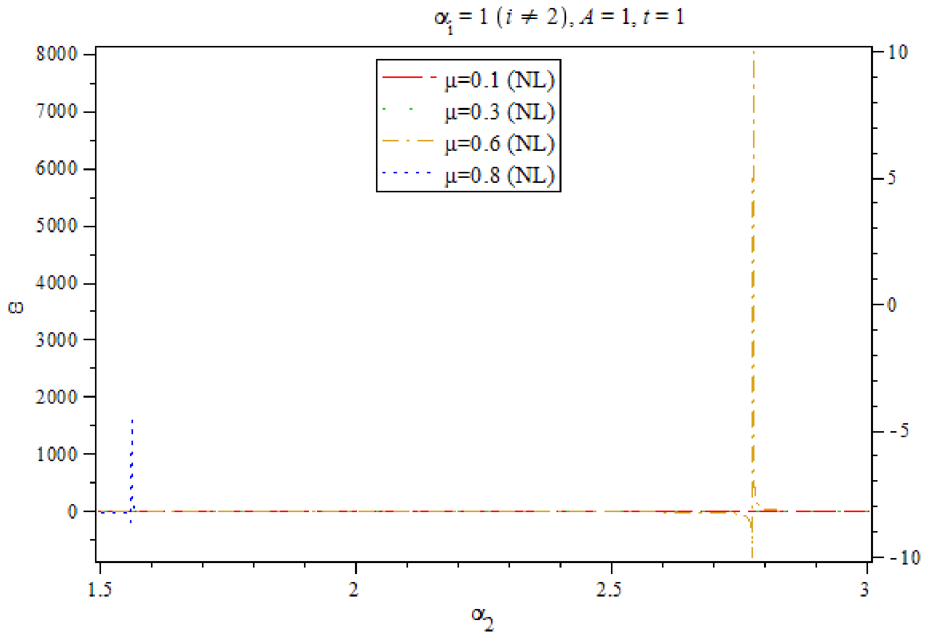

Fig.7. The effects of nonlocal parameter μ on the free vibration response of nano beam for $\alpha_i = 1 (i = 1 \ldots 9)$, $A = 1$ and $t = 1$

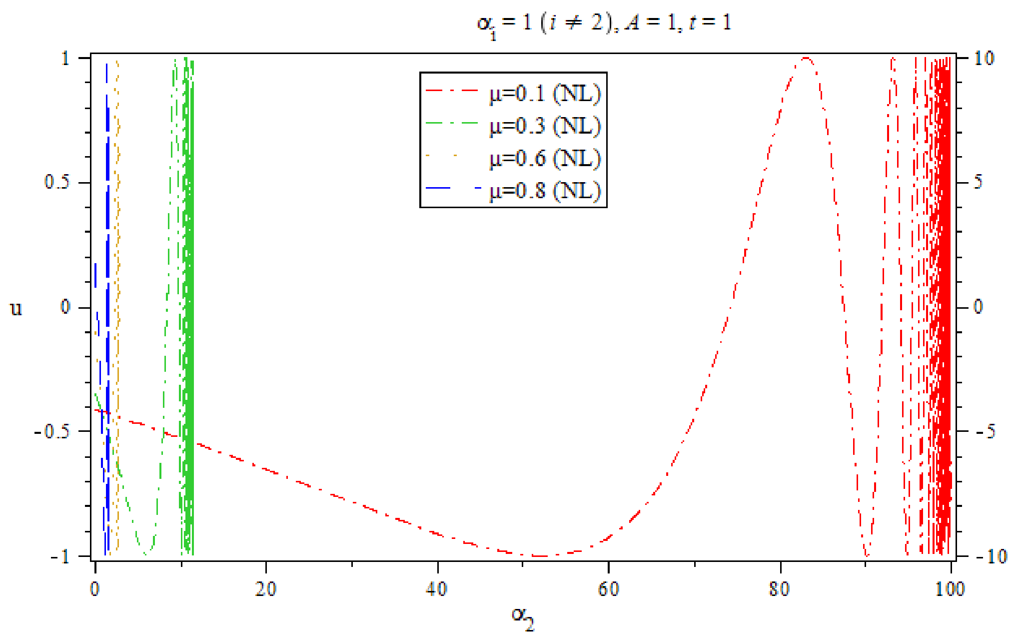

Fig.8. The effects of nonlocal parameter μ on the free vibration response of nano beam for $\alpha_i = 1 (i = 1 \mathrm{L}\ 9)$, $A = 1$ and $t = 1$

Figs. 9 and 10 show the effects of α_3 parameter on the frequency and the free vibration response of nano beam for constant values of other parameters. According to Eq. (29), parameter of α_3 dependent on the viscous damping constant (B). According to Fig. 9, it can be seen that in all of values of μ, increasing in the viscous damping constant, reduce the natural frequency of the system that this decrease rate is higher by increasing of nonlocal parameter. Fig. 10 also show that by increasing in values of nonlocal parameter μ, the number of oscillations of the harmonic response of the piezo-actuated nano increase.

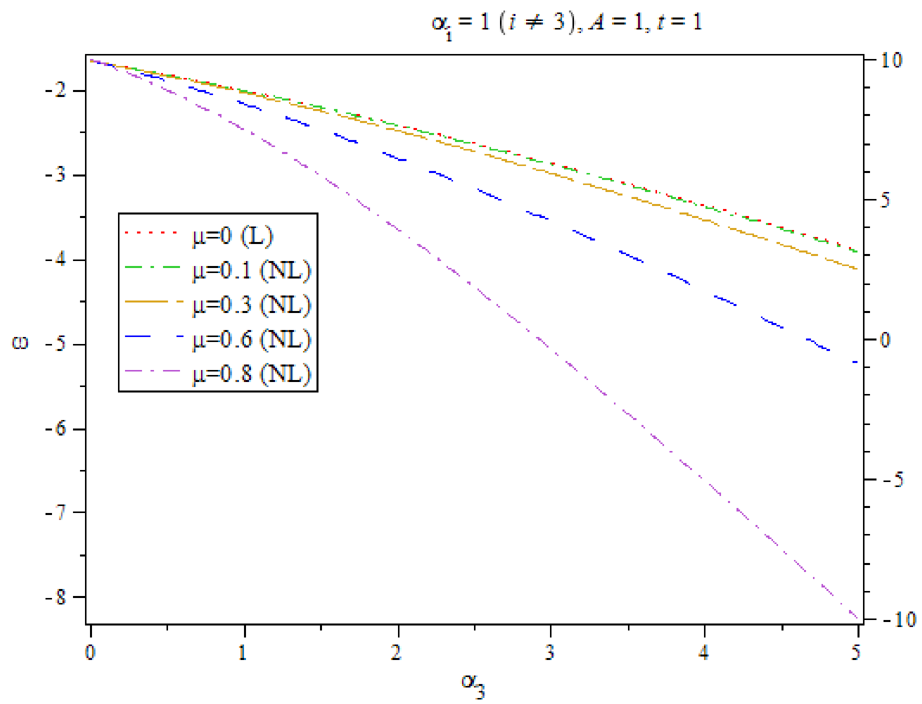

Fig.9. The effects of nonlocal parameter μ on the natural frequency of nano beam via α_3

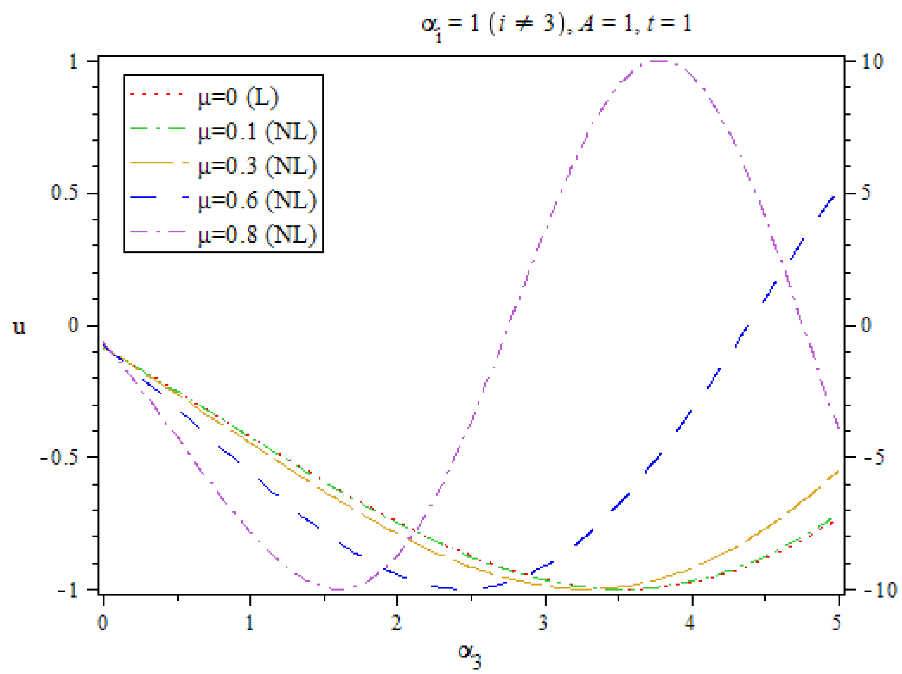

Fig.10. The effects of nonlocal parameter μ on the free vibration response of nano beam via α_3

Figs. 11 and 12 show the effects of α_5 parameter on the frequency and the free vibration response of nano beam for constant values of other parameters. According to Eq. (29), parameter of α_5 dependent on the $(EI)_{eq}$

. According to Fig. 11, it can be seen that in all of values of μ, as the viscous damping constant, increasing in $(EI)_{eq}$, reduce the natural frequency of the system that this decrease rate is higher by increasing of nonlocal parameter. Fig. 12 also show that by increasing in values of nonlocal parameter μ, the number of oscillations of the harmonic response of the piezo-actuated nano increase.

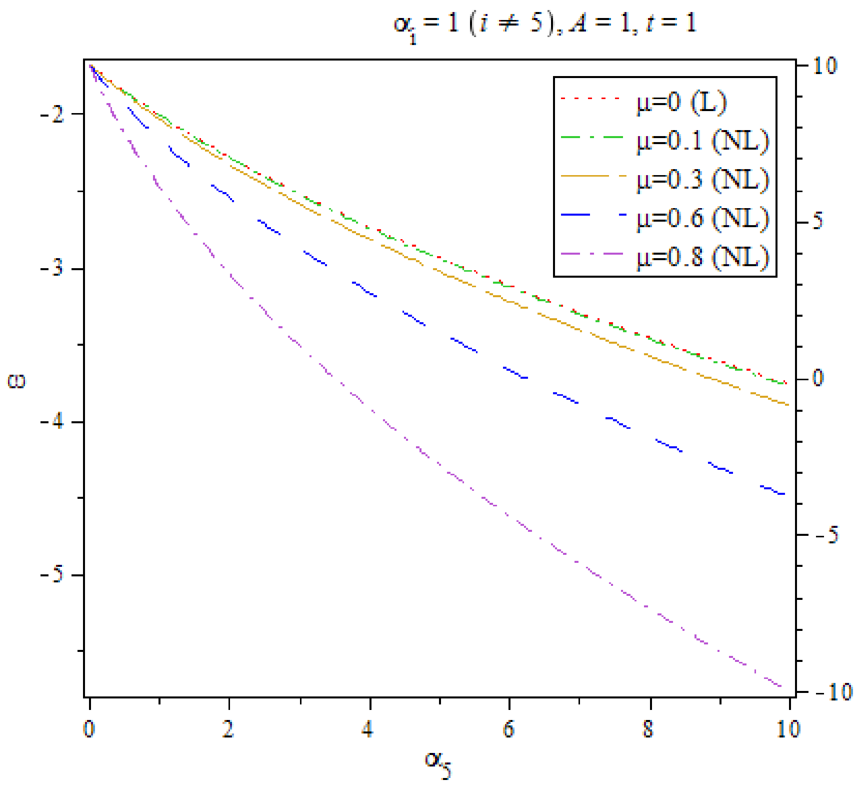

Fig.11. The effects of nonlocal parameter μ on the natural frequency of nano beam via α_5

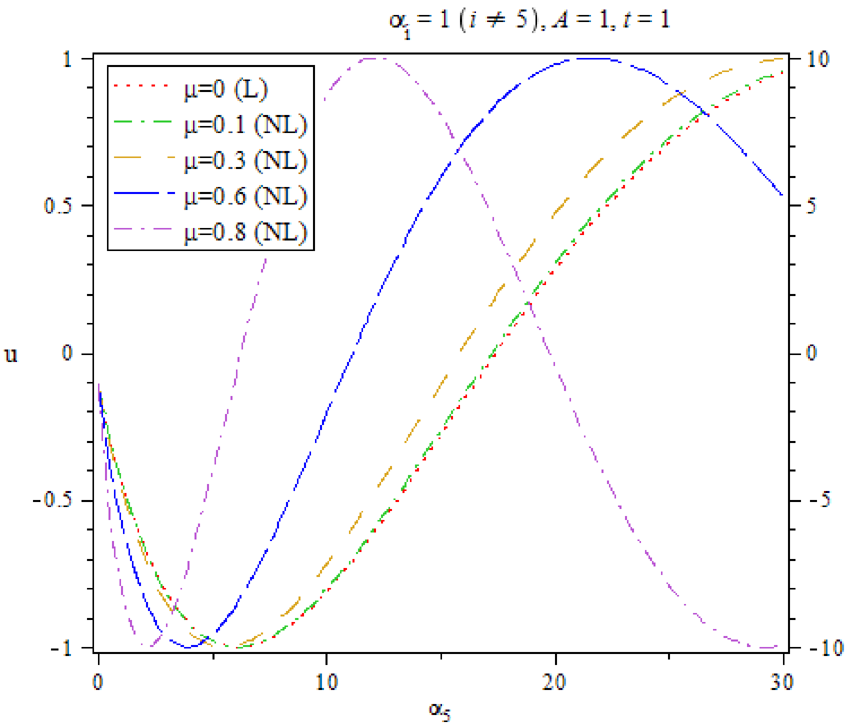

Fig.12. The effects of nonlocal parameter μ on the free vibration response of nano beam via α_5

Figs. 13 and 14 show the effects of α_6 parameter on the frequency and the free vibration response of nano beam for constant values of other parameters. According to Eq. (29), parameter of α_6 dependent on the axial force of the piezoelectric voltage (F_p). According to Fig. 13, it can be seen that in all of values of μ , increasing in the piezoelectric voltage, reduce the natural frequency of the system that this decrease rate is higher by increasing of nonlocal parameter. Fig. 14 also show that by increasing in values of nonlocal parameter μ , the number of oscillations of the harmonic response of the piezo-actuated nano increase.

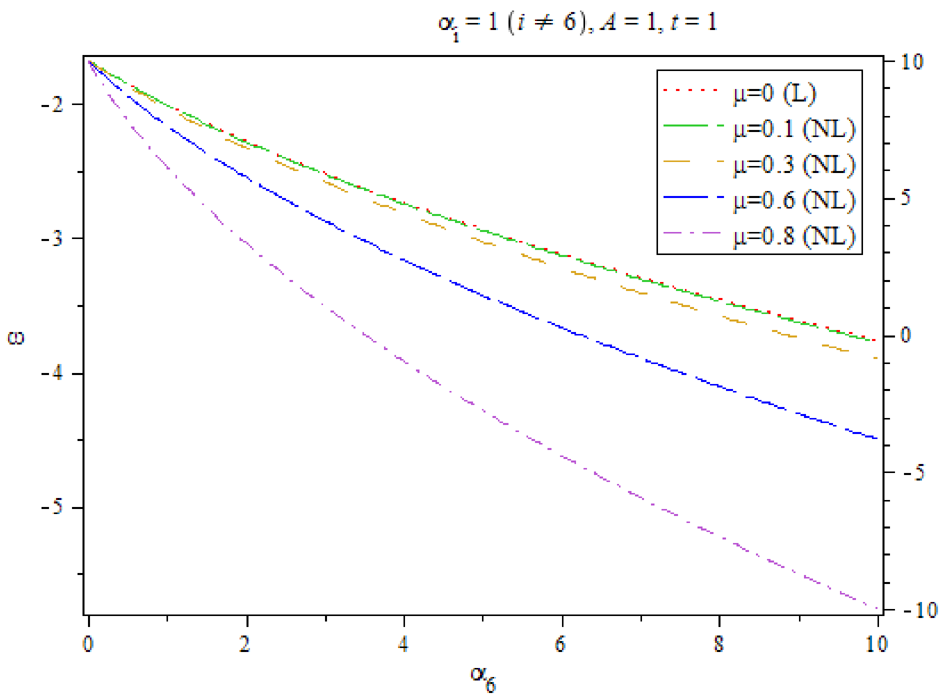

Fig.13. The effects of nonlocal parameter μ on the natural frequency of nano beam via α_6

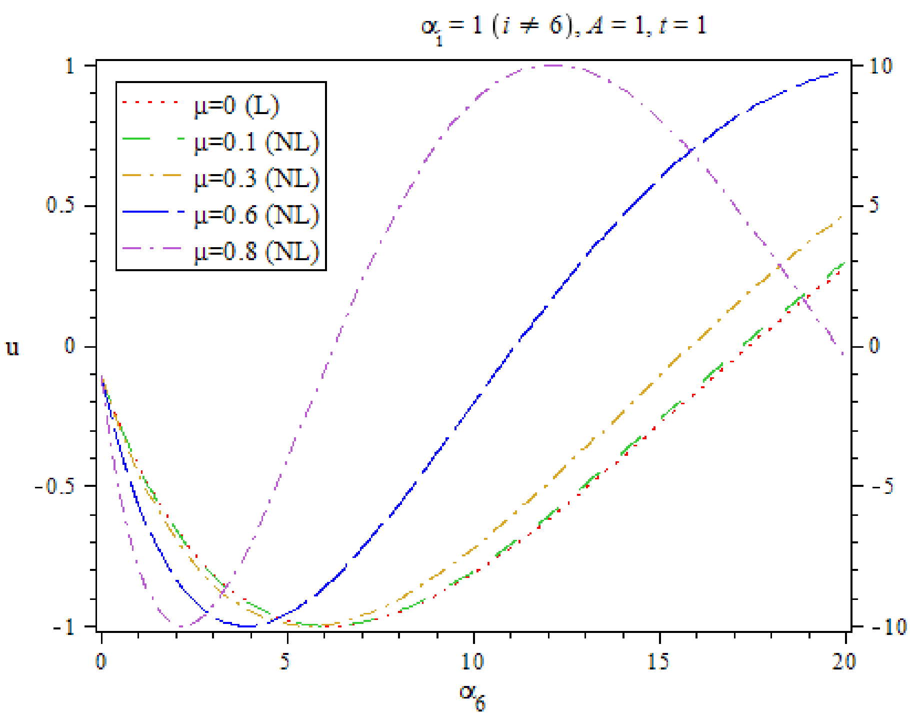

Fig.14. The effects of nonlocal parameter μ on the free vibration response of nano beam via α_6

Figs. 15 and 16 show the effects of α_6 parameter on the frequency and the free vibration response of nano beam for constant values of other parameters. According to Eq. (29), parameter of α_6 dependent on the axial force of the nano beam (F_a). According to Fig. 15, it can be seen that in all of values of μ, increasing in the axial force, reduce the natural frequency of the system that this decrease rate is higher by increasing of nonlocal parameter. Fig. 16 also show that by increasing in values of nonlocal parameter μ, the number of oscillations of the harmonic response of the piezo-actuated nano increase.

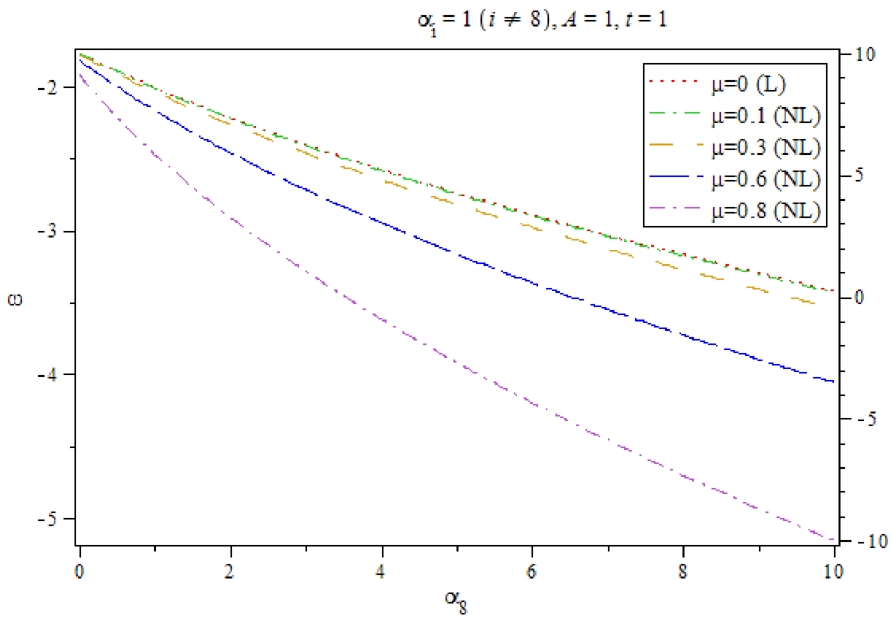

Fig.15. The effects of nonlocal parameter μ on the natural frequency of nano beam via α_8

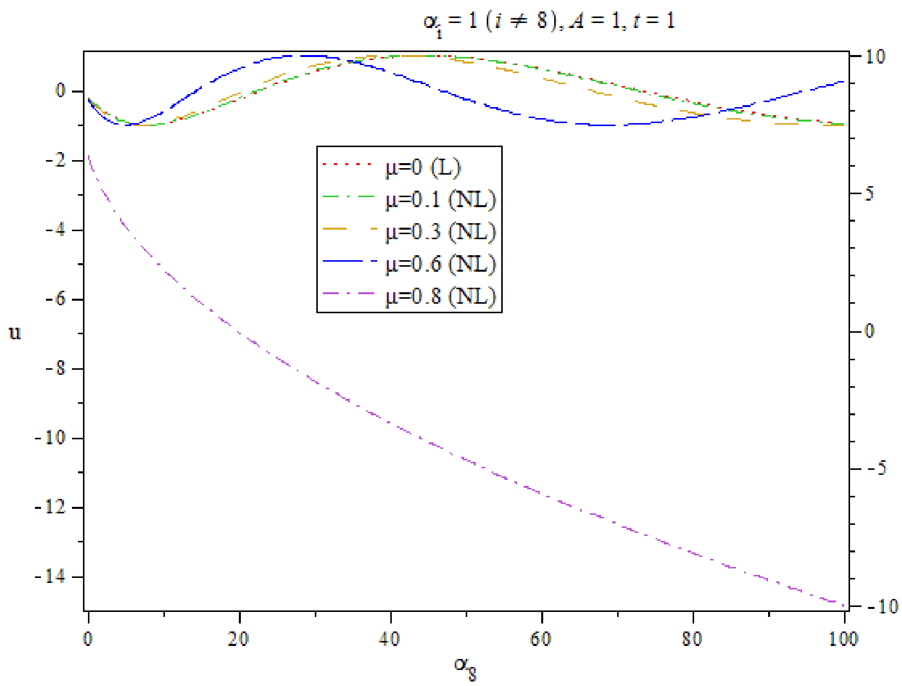

Fig.16. The effects of nonlocal parameter μ on the free vibration response of nano beam via α_8

Conclusions

In this study, nonlinear vibration analysis of a parametrically excited piezoelectric nano beam subjected to DC and AC voltages is investigated for biological sensor applications on the basis of the non-local continuum theory. Equations of the motion and boundary conditions of the nano beam are obtained by implementation of Hamilton's principle and the Galerkin approach. Hamiltonian solution namely Frequency-Amplitude approach is used for natural frequencies and mode shapes as a function of the piezo-layered nano beam characteristic non-local size scale parameter. It was seen that natural frequencies decrease as the non-local parameter increases and this reduction is more prominent for higher natural frequencies. Besides, by increasing in values of nonlocal parameter μ, the number of oscillations of the harmonic response of the piezo-sensor nano increase. The size effects on the vibration behavior (frequency and harmonic response) of the beam have been studied and it is found that the non-local parameter has significant effects on the free vibration of system.

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Conflicts of Interest

The authors report no conflict of interest.

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