On another form of the energy density for electromagnetic fields and its relation to the electrokinetic field

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Abstract: By virtue of the existence so called electrokinenic field encountered in electromagnetic phenomena, a new form of the energy density of the electromagnetic field is obtained.

1. Introduction

Oppenheimer and Ahluwalia [1-3] independently found that source-free Maxwell equations have negative energy solutions and zero-energy solutions. You can either say that negative energy decisions, and decisions with equally disappearing energy content should be discarded. Or take into account that the usual "quadratic expression in **E** and **H**" for the energy density of the electromagnetic field is not complete. This problem was partially solved by one of the authors of this article in [4]. Here we argue that it is the latter that takes place, that is, the well-known expression for the energy density of the electromagnetic field is not complete, and we offer an explicit construction for such a modified expression for the energy density, taking into account at that time still unknown to the authors [1-3, 4] the existence of a special type of electric field, theoretically discovered by O. Jefimenko [5], namely, the *electrokinetic field*.

If $\phi_L(\mathbf{p})$ и $\phi_R(\mathbf{p})$ represent the massless (1, 0) и (0, 1) fields respectively [1], then the source-free momentum-space Maxwell equation can be written as (see, e.g. [4])[1]

$$\left(\mathbf{J}\cdot\mathbf{p}+p^0\right)\phi_L(\mathbf{p})=0, \tag{1}$$

$$\left(\mathbf{J}\cdot\mathbf{p}+p^0\right)\phi_R(\mathbf{p})=0, \tag{2}$$

where $\mathbf{J}$ are the 3×3 spin -1 angular momentum matrices

$$J_x=\begin{pmatrix}0&0&0\\0&0&-i\\0&i&0\end{pmatrix},\quad J_y=\begin{pmatrix}0&0&i\\0&0&0\\-i&0&0\end{pmatrix},\quad J_z=\begin{pmatrix}0&-i&0\\i&0&0\\0&0&0\end{pmatrix}. \tag{3}$$

Oppenheimer [2] and Ahluwalia [1, 3] independently noted that in order for nontrivial

solutions of Eqs. (1) and (2) to exist, one must have

$$p^0=\pm|\mathbf{p}|,\quad p^0=0. \tag{4}$$

These "dispersion relations" follow from the condition $\mathrm{Det}\left(\mathbf{J}\cdot\mathbf{p}\pm p^0\right)=0$.

This situation immediately causes two problems: (i) negative energy solutions exist, and (ii) the equations support zero-energy solutions. One can either say that negative energy siolutions, and siolutions with equally disappearing energy content should be discarded. Or, if we consider that the usual "quadratic expression in **E**

[1] Maxwell's equations in the configuration space follow on setting $\mathbf{p}=i\nabla$, $p^0=i\frac{\partial}{\partial t}$ $\phi_R(x)$ and creating appropriate linear combinations $\phi_L(x)$ to obtain the fields $\mathbf{E}$ and $\mathbf{H}$.

and **H**" for the energy density of the electromagnetic field is not complete. Here we argue that the latter takes place, providing an explicit construction for such a specified modified expression for the energy density.

Recall the generally accepted way to obtain electromagnetic energy density field in vacuum [6].

To obtain the energy density of the electromagnetic field and the flux density of electromagnetic energy, Landau and Lifshitz (see § 34, p. 76 of [6]) used two Maxwell equations:

$$\nabla \times \mathbf{H} = \frac{4}{\pi c}\mathbf{j} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} \tag{5}$$

и

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t}\,. \tag{6}$$

Landau and Lifshitz multiplied both sides (5) by **E** and both sides (6) by **H** and combined the resulting equations:

$$\frac{1}{c}\mathbf{E}\cdot\frac{\partial \mathbf{E}}{\partial t} + \frac{1}{c}\mathbf{H}\cdot\frac{\partial \mathbf{H}}{\partial t} = -\frac{4}{\pi c}\mathbf{j}\cdot\mathbf{E} - (\mathbf{H}\cdot[\nabla\times\mathbf{E}] - \mathbf{E}\cdot[\nabla\times\mathbf{H}])\,, \tag{7}$$

then, using the well-known vector analysis formula, get:

$$\frac{\partial}{\partial t}\left(\frac{E^2+H^2}{8\pi}\right) = -\mathbf{j}\cdot\mathbf{E} - \nabla\cdot\mathbf{S}\,, \tag{8}$$

where the vector

$$\mathbf{S} = \frac{c}{4\pi}[\mathrm{E}\times\mathbf{H}] \tag{9}$$

is called the *Poynting vector*. Then (8) is integrated over the volume and the Gauss theorem is applied to the second term on the right-hand side.

$$\frac{\partial}{\partial t}\int\frac{E^2+H^2}{8\pi}dV = -\int\mathbf{j}\cdot\mathbf{E}\,dV - \oint\mathbf{S}\cdot d\mathbf{f}. \tag{10}$$

If the integral, Landau and Lifshitz wrote further, spreads throughout the all space, then the surface integral is zero (the field is zero at infinity). Then one can express the integral $\int\mathbf{j}\cdot\mathbf{E}\,dV$ as the sum как сумму $\sum q\mathbf{v}\cdot\mathbf{E}$ over all charges and substitute from (17.7) [6]

$$\sum q\mathbf{v}\cdot\mathbf{E} = \frac{d}{dt}\mathcal{E}_{\mathrm{kin}}\,.$$

As a result, Landau has:

$$\frac{d}{dt}\left\{\int\frac{E^2+H^2}{8\pi}dV + \sum\mathcal{E}_{\mathrm{kin}}\right\} = 0. \tag{11}$$

Thus, Landau and Lifshitz concluded that for a closed system consisting of an electromagnetic field and the particles present in it, the value in brackets in this equation is constant with respect to time. The second term in this expression is kinetic energy (including the rest energy of all particles, of course), so the first term is the energy of the field itself. Therefore, one could name the quantity

$$w = \frac{E^2 + H^2}{8\pi} \tag{12}$$

energy density of the electromagnetic field. Obviously, it is impossible to reconcile such a definition of energy density with such a configuration of fields when w is equal to zero at some point, and the fields **E** and **H** are not equal to zero at the same point.

Here, however, we must make some important comments:

a) Landau and Lifshitz used the transition $\frac{\partial}{\partial t}\int(\dots) \to \frac{d}{dt}\int(\dots)$ for a field too freely, without any clarification of this mathematical operation.

b) Landau and Lifshitz (see § 31 of [6]) stated that the surface integral $\oint \mathbf{S}\cdot d\mathbf{f}$ vanishes at infinity, since the field is equal to zero at infinity. But in this case, the field of radiation, which can go to infinity, is implicitly neglected. other words, one cannot go from (10) to (11) without imposing some additional conditions that prevent this. To be more specific, we again turn to [6] (the first footnote in § 34 of [6]): *Here we also assume that the electromagnetic field of the system also disappears at infinity. This means that if the system emits electromagnetic waves, it is assumed that special "reflecting walls" prevent the exit of these waves to infinity*.

We make an important note here: In classical electrodynamics, it is assumed that the law of conservation of energy is an absolute law and in order to satisfy this law, we must, in general, take into account the possible change in the energy of these "reflecting walls", which can occur as a result of energy exchange between these "walls" and the "particle + field" system.

However, we do not know the mathematically correct way to take this energy into account in formula (11) without an exact knowledge of the "nature" of "reflecting walls". In this case, one cannot obtain the exact law of conservation of energy using the concept of "reflecting walls". In other words, to obtain the exact law of conservation of energy, these "walls" should not be introduced, but we must assume that the surface integral $\oint \mathbf{S}\cdot d\mathbf{f}$ *does not vanish* at infinity. But in this case, equation (10) turns into a trivial equality, which, although it satisfies the exact law of energy conservation, cannot be used to draw any conclusion about a specific mathematical form of the energy density of the electromagnetic field.

2. The relationship of another form of energy density with Oppenheimer-Aluwalia the zero-energy solutions of the Maxwell equations

However, there is a way to get an explicit form of the energy density of the electromagnetic field. We turn to our articles [7,8], where we proved that the electromagnetic field should be represented by two *independent* parts:

$$\mathbf{E} = \mathbf{E}_0 + \mathbf{E}^* = \mathbf{E}_0\left(\mathbf{r} - \mathbf{r}_q(t)\right) + \mathbf{E}^*(\mathbf{r}, t), \tag{13a}$$

where $\mathbf{E}_0$ is the electric charge q field, and $\mathbf{E}^*$ is the well-known so-called free (*radiated*) electric field, and now taking into account the existence of the *electrokinetic field* $\mathbf{E}_k$ [5][2]

$$\mathbf{E} = \mathbf{E}_0 + \mathbf{E}^* + \mathbf{E}_k = \mathbf{E}_0\left(\mathbf{r} - \mathbf{r}_q(t)\right) + \mathbf{E}^*(\mathbf{r}, t) + \mathbf{E}_k(\mathbf{r}, t), \tag{13}$$

[2] The electric field created by time-variable currents is *very* different from all other fields encountered in electromagnetic phenomena. So O. D. Jefimenko in [5], taking into account that the cause of this field is a motion of electric charges (current), gives to it the special name the *electrokinetic field*. $\mathbf{E}_k = -\frac{1}{4\pi\varepsilon_0 c^2}\int \frac{1}{r}\left[\frac{\partial \mathbf{J}}{\partial t}\right] dV'$ [5], where the square brackets in this equation are the retardation symbol indicating that the quantities between the brackets are to be evaluated for the time $t' = t - r/c$, where t is the time for which $\mathbf{E}_k$ is evaluated.

$$\mathbf{H} = \mathbf{H}_0 + \mathbf{H}^* = \mathbf{H}_0\left(\mathbf{r} - \mathbf{r}_q(t)\right) + \mathbf{H}^*(\mathbf{r}, t). \tag{14}$$

Here we note that quasistatic components, such as $\mathbf{E}_0$ and $\mathbf{H}_0$, depend only on the distance between the observation point and the position of the source at the time of observation, while time-varying fields, such as $\mathbf{E}^*$, $\mathbf{E}_k$ and $\mathbf{H}^*$, clearly depend on observation points and observation time.

Now we rewrite equations (5) and (6) as formulas (45) and (46) from our mentioned above article [8]. Please note that here the time derivatives of the fields are not partial, but *total*:

$$\nabla \times \mathbf{H} = \frac{4}{\pi c}\mathbf{j} + \frac{1}{c}\frac{d\mathbf{E}}{dt}, \tag{15}$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{d\mathbf{H}}{dt}, \tag{16}$$

where one can calculate the total time derivative of any value of the vector field **E** (or **H**) according to the following rule:

$$\frac{d\mathbf{E}}{dt} = \frac{\partial \mathbf{E}^*}{\partial t} + \frac{\partial \mathbf{E}_k}{\partial t} - \left(\sum_i \mathbf{V}_i \cdot \nabla\right)\mathbf{E}_0, \tag{17}$$

here $\mathbf{V}_i$ are the particle velocities at the same moment of observation.[3]

The mutual independence of the fields $\{\}_0$, $\mathbf{E}_k$ and $\{\}^*$ allows us to rewrite equations (15) and (16) (taking into account relations (13), (14) and (17)) in the form of two unconnected pairs of differential equations:

$$\nabla \times \mathbf{H}^* = \frac{4}{\pi c}\mathbf{j} + \frac{1}{c}\frac{\partial(\mathbf{E}^* + \mathbf{E}_k)}{\partial t} \tag{18}$$

$$\nabla \times (\mathbf{E}^* + \mathbf{E}_k) = -\frac{1}{c}\frac{\partial \mathbf{H}^*}{\partial t}, \tag{19}$$

and

$$\nabla \times \mathbf{H}_0 = \frac{4}{\pi c}\mathbf{j} - \frac{1}{c}\left(\sum_i \mathbf{V}_i \cdot \nabla\right)\mathbf{E}_0, \tag{20}$$

$$\nabla \times \mathbf{E}_0 = \frac{1}{c}\left(\sum_i \mathbf{V}_i \cdot \nabla\right)\mathbf{H}_0. \tag{21}$$

Let us finally repeat the calculations of Landau and Lifshitz (see above), but now, taking into account equations (15) and (16) and without imposing conditions like "reflecting walls".

Multiply both sides of (15) by **E** and both sides of (16) by **H** and combine the resulting equations. Then we have:

$$\frac{1}{c}\mathbf{E} \cdot \frac{d\mathbf{E}}{dt} + \frac{1}{c}\mathbf{H} \cdot \frac{d\mathbf{H}}{dt} = -\frac{4}{\pi c}\mathbf{j} \cdot \mathbf{E} - (\mathbf{H} \cdot [\nabla \times \mathbf{E}] - \mathbf{E} \cdot [\nabla \times \mathbf{H}]). \tag{22}$$

Using the rule (17) and the well-known vector analysis formula, we obtain:

[3] Note (see [7] and [8]) that, unlike the field $\mathbf{E}_k$ and fields $\{\}^*$, the fields $\{\}_0$ are not retarded.

$$\frac{1}{c}\mathbf{E}\cdot\left\{\frac{\partial(\mathbf{E}^*+\mathbf{E}_k)}{\partial t}-\left(\sum_i \mathbf{V}_i\cdot\nabla\right)\mathbf{E}_0\right\}+\frac{1}{c}\mathbf{H}\cdot\left\{\frac{\partial\mathbf{H}^*}{\partial t}-\left(\sum_i \mathbf{V}_i\cdot\nabla\right)\mathbf{H}_0\right\} = -\frac{4}{\pi c}\mathbf{j}\cdot\mathbf{E}-\nabla\cdot[\mathbf{E}\times\mathbf{H}]\,. \tag{23}$$

Then, taking into account relations (13), (14), and also that

$$\frac{d(\mathbf{E}^*+\mathbf{E}_k)}{dt}=\frac{\partial(\mathbf{E}^*+\mathbf{E}_k)}{\partial t} \quad u \quad \frac{d\{\}_0}{dt}=-\left(\sum_i \mathbf{V}_i\cdot\nabla\right)\{\}_0\,,$$

and, finally, after some transformations we have:

$$\frac{\partial}{\partial t}\left(\frac{E^2+H^2}{8\pi}\right)+\frac{d}{dt}\left(\frac{2(\mathbf{E}^*+\mathbf{E}_k)\cdot\mathbf{E}_0+2\mathbf{H}^*\cdot\mathbf{H}_0+E_0^2+H_0^2}{8\pi}\right) = -\mathbf{j}\cdot\mathbf{E}-\nabla\cdot\left(\frac{c}{4\pi}[\mathbf{E}\times\mathbf{H}]\right)\,. \tag{24}$$

Now we can integrate this expression over a volume (taking into account the relation $q\mathbf{v}\cdot\mathbf{E}=\frac{d}{dt}\mathcal{E}_{\text{kin}}$):

$$\frac{\partial}{\partial t}\int\frac{(\mathbf{E}^*+\mathbf{E}_k)^2+H^{*2}}{8\pi}dV+\frac{d}{dt}\left(\int\frac{2(\mathbf{E}^*+\mathbf{E}_k)\cdot\mathbf{E}_0+2\mathbf{H}^*\cdot\mathbf{H}_0+E_0^2+H_0^2}{8\pi}dV+\sum\mathcal{E}_{\text{kin}}\right) = -\int\nabla\cdot\left(\frac{c}{4\pi}[\mathbf{E}\times\mathbf{H}]\right)dV. \tag{25}$$

We now take these integrals over all space and apply the Gauss theorem to the right-hand side of (25). In this case, given that the fields $\{\}_0$ associated with the particles vanish at infinity, we obtain:

$$-\int\nabla\cdot\left(\frac{c}{4\pi}[\mathbf{E}\times\mathbf{H}]\right)dV\to\oint\left(\frac{c}{4\pi}[(\mathbf{E}^*+\mathbf{E}_k)\times\mathbf{H}^*]\right)\cdot d\mathbf{f} = -\int\nabla\cdot\left(\frac{c}{4\pi}[(\mathbf{E}^*+\mathbf{E}_k)\times\mathbf{H}^*]\right) \tag{26}$$

It is easy to verify, taking into account equations (18) and (19), that the last integral in (26) and the first integral in (25) are equal to each other. Then (25) becomes:

$$\frac{d}{dt}\left(\int\frac{2(\mathbf{E}^*+\mathbf{E}_k)\cdot\mathbf{E}_0+2\mathbf{H}^*\cdot\mathbf{H}_0+E_0^2+H_0^2}{8\pi}dV+\sum\mathcal{E}_{\text{kin}}\right)=0\,. \tag{27}$$

Therefore, we can call the value

$$w=\frac{2(\mathbf{E}^*+\mathbf{E}_k)\cdot\mathbf{E}_0+2\mathbf{H}^*\cdot\mathbf{H}_0+E_0^2+H_0^2}{8\pi} \tag{28}$$

the energy density of the electromagnetic field.

We note once again that one can never get the so-called "*zero or negative Oppenheimer-Aluwalia energy solutions*" [1-3] from the generally accepted form of electromagnetic energy density

$$w=\frac{E^2+H^2}{8\pi}\,, \tag{29}$$

because for real fields this quantity is *always* positive and can be equal to zero only when the fields **E** and **H** are equal to zero *simultaneously*.

But from our *new representation* of the electromagnetic field energy density

$$w = \frac{2(\mathbf{E}^* + \mathbf{E}_k)\cdot\mathbf{E}_0 + 2\mathbf{H}^*\cdot\mathbf{H}_0 + E_0^2 + H_0^2}{8\pi}\ , \tag{30}$$

it is easy to see that the fields $\mathbf{E}_k$, $\mathbf{E}^*$ and $\mathbf{E}_0$ *can* have *mutually different* signs, because these fields $\mathbf{E}_k$, $\mathbf{E}^*$ and $\mathbf{E}_0$ are *different*. This means that we can have the following relationship:

$$2(\mathbf{E}^* + \mathbf{E}_k)\cdot\mathbf{E}_0 + 2\mathbf{H}^*\cdot\mathbf{H}_0 < 0\,, \tag{31}$$

and, in turn, we can have a configuration of nonzero fields for which w is zero:

$$2(\mathbf{E}^* + \mathbf{E}_k)\cdot\mathbf{E}_0 + 2\mathbf{H}^*\cdot\mathbf{H}_0 = -\left(E_0^2 + H_0^2\right). \tag{32}$$

In fact, it is enough that the fields $(\mathbf{E}^* + \mathbf{E}_k)$ and $\{\ \}_0$ satisfy the equations:

$$|(\mathbf{E}^* + \mathbf{E}_k)| = \frac{-|\mathbf{E}_0|}{2\cos\alpha}\ u\ |\mathbf{H}^*| = \frac{-|\mathbf{H}_0|}{2\cos\alpha}\,, \tag{33}$$

где α – an angle between vectors $(\mathbf{E}^* + \mathbf{E}_k)$ и $\{\ \}_0$ with the following limits:

$$\frac{\pi}{2} < \alpha < \pi + \frac{\pi}{2}\,. \tag{34}$$

From formulas (30) and (31), we can also see that there are negative energy solutions (compare with remark (i) after equation (4)).

3. Discussion

In this work, we used the concept of the Poynting vector, but the concept of the field momentum density was not used. Let us elucidate our point of view:

On the one hand, we know from the *generally accepted* classical electrodynamics that the Poynting vector is proportional to the momentum density of the electromagnetic field. On the other hand, there are paradoxes associated with the Poynting vector, and they are well known. For example, in the work of one of the authors of this article they say [7]: if the charge *Q* vibrates by some mechanical way along the *X*-axis, then the value of *w* (which is a point function, such as $|\mathbf{E}|$) on the same axis will also oscillate. The question arises: *how does the test charge q at the observation point, lying at some fixed distance from the charge Q along the extension of the X-axis, "know" about the vibration of the charge Q?* In other words, we have a rather *strange* situation: the Poynting vector S = c4π [E × H] is equal to zero along this axis (since **H** is equal to zero along this line), but the energy and momentum obviously "pass" from point to point along this axis. Другими словами, мы имеем довольно странную ситуацию: вектор Пойнтинга **S** = *c*4[**E**×**H**] In other words, we have a rather strange situation: the Poynting vector **S** = *c4π* [**E** × **H**] is equal to zero along this axis (since **H** is equal to zero along this line), but the energy and momentum obviously "pass" from point to point along this axis. This means that we cannot be sure that using the *new* definition of energy density will allow us to use the *old* definition of momentum density. This problem, in our opinion, requires very thorough research. Other quantities of classical electrodynamics, such as the electromagnetic field tensor, electromagnetic energy-momentum tensor, etc., can (and possibly should) also change their physical values. In fact, in recent years a significant number of works have been published that directly state: classical electrodynamics should be *substantially* revised[4]. To be more specific, let us end this

[4] See a brilliant review of these works, "An Essay on Non-Maxwell Theories of Electromagnetism." V.V. Dvoeglazov [9].

article with the words of R. Feynman [10]: "… *this tremendous edifice (classical electrodynamics), which is such a beautiful success in explaining so many phenomena, ultimately falls on its face. When you follow any of our physics too far, you find that it always gets into some kind of trouble… the failure of the classical electromagnetic theory… Classical mechanics is a mathematically consistent theory; it just doesn't agree with experience. It is interesting, though, that the classical theory of electromagnetism is an unsatisfactory theory all by itself. There are difficulties associated with the ideas of Maxwell's theory which are not solved by and not directly associated with quantum mechanics…*" . As well as by words of A. O. Barut [11] "… *Electrodynamics and the classical theory of fields remain very much alive and continue to be the source of inspiration for much of the modern research work in new physical theories.*"

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