On new classes of some nano closed sets

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Abstract

The aim of this paper is to introduce a new class of sets called N$^*\mu$-closed sets in Nano topological spaces and to study some of its basic properties. As applications of N$^*\mu$-closed sets, we introduce T$_{N^*\mu}$-spaces, $_g$T$_{N^*\mu}$-spaces and $_\alpha$T$_{N^*\mu}$-spaces. Moreover, we obtain certain new characterizations for the T$_{N^*\mu}$-spaces, $_g$T$_{N^*\mu}$-spaces and $_\alpha$T$_{N^*\mu}$-spaces.

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Introduction

Lellis Thivagar and Carmel Richard [10] introduced and studied Nano semi-open, Nano α-open, Nano-preopen and Nano regular open respectively. Revathy and Illango [14] introduced and studied Nano β-open sets. Bhuvaneswari and Mythili Gnanapriya [2] introduced Nano generalised closed sets. Bhuvaneshwari and Ezhilarasi [4], Thanga Nachiyar and Bhunaneswari [16] defined Nano generalized α-closed sets and Nano α generalized closed sets respectively. S. Ganesan et al [6] studied Nano *g-closed sets. The aim of this paper, we introduce and study some basic properties of N$^*\mu$-closed sets. As applications of N$^*\mu$-closed sets, we introduce and study new spaces, namely T$_{N^*\mu}$-spaces, $_g$T$_{N^*\mu}$-spaces and $_\alpha$T$_{N^*\mu}$-spaces. Moreover, we obtain their properties and characterizations.

1 PRELIMINARIES

1.1 Definition

[10] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let X $\subseteq$ U.

1. The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by L$_R(X)$.
   $Thatis, L_R(X) = \bigcup_{x\in U}$ {R(X): R(X) $\subseteq$ X} where R(x) denotes the equivalence class determined by X.
2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X

with respect to R and it is denoted by $U_R(X)$.
$Thatis, U_R(X) = \bigcup_{x\in U}$ {R(X): R(X) ∩ X ≠ ϕ}

3. The boundary region of X with respect to R is the set of all objects, which can be neither in nor as not-X with respect to R and it is denoted by $B_R(X).Thatis, B_R(X) = U_R(X) - L_R(X)$

1.2 Property

[10] If (U, R) is an approximation space and X, Y ⊆ U, then

1. $L_R(X) \subseteq X \subseteq U_R(X)$.
2. $L_R(\phi) = U_R(\phi) = \phi$, $L_R(U) = U_R(U) = U$.
3. $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$.
4. $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$.
5. $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$.
6. $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$.
7. $L_R(X) \subseteq L_R(Y) and U_R(X) \subseteq U_R(Y) whenever X \subseteq Y$.
8. $U_R(X^c) = [L_R()]^c$ and $L_R(^c) = [U_R()]^c$.
9. $U_R(U_R(X)) = L_R(U_R(X)) = U_R(X)$.
10. $L_R(L_R(X)) = U_R(L_R(X)) = L_R(X)$.

1.3 Definition

[10] Let U be an universe, R be an equivalence relation on U and $\tau_R(X)$ ={U, ϕ , $L_R(X), U_R(X), B_R(X)\} where X \subseteq$ U. Then by Property 1.2, $\tau_R(X)$ satisfies the following axioms

1. U , $\phi \epsilon \tau_R(X)$.
2. The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.
3. The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

   Then $\tau_R(X)$ is called the Nano topology on U with respect to X.

   The space (U, $\tau_R(X)$) is the Nano topological space. The elements of are called Nano open sets.

1.4 Definition

[10]

If (U, $\tau_R(X)$) is the Nano topological space with respect to X where X $\subseteq$ U and if M $\subseteq$ U, then

1. The Nano interior of the set M is defined as the union of all Nano open subsets contained in M and it is denoted by NInte(M). That is, NInte(M) is the largest Nano open subset of M.
2. The Nano closure of the set M is defined as the intersection of all Nano closed sets containing M and it is denoted by NClo(M). That is, NClo(M) is the smallest Nano closed set containing M.

1.5 Definition

A subset M of a space (U, $\tau_R(X)$) is called:

1. Nano α-open set [10] if M $\subseteq$ Ninte(Nclo(Ninte(M))).
2. Nano semi-open set [10] if M $\subseteq$ Nclo(Ninte(M)).
3. Nano pre-open set [10] if M $\subseteq$ Ninte(Nclo(M)).
4. Nano β-open set [14] if M $\subseteq$ Nclo(Ninte(Nclo(M))).
5. Nano regular-open set [10] if M = Ninte(Nclo(M)).
6. Nano π-open set [1] if finite union of Nano regular-open set.

The complements of the above mentioned Nano open sets are called their respective Nano closed sets.

The Nano α-closure [7] (resp. Nano semi-closure [4, 5], Nano pre-closure [3], Nano semi-pre-closure) of a subset M of U, denoted by Nαclo(M) (resp.Nsclo(M), Npclo(M), Nβclo(M)) is defined to be the intersection of all Nano α-closed (resp. Nano semi-closed, Nano pre closed, Nano semi pre closed) sets of (U, $\tau_R(X)$) containing M.

1.6 Definition

A subset M of a space (U, $\tau_R(X)$) is called:

1. a Nano generalized closed (briefy Ng-closed) set [2] if Nclo(M) $\subseteq$ T whenever M $\subseteq$ T and T is Nano open in (U, $\tau_R(X)$).
2. a Nano $\hat{g}$-closed (briefly $\hat{g}$-closed) set [8] if Nclo(M) $\subseteq$ T whenever M $\subseteq$ T and T is Nano semi-open in (U, $\tau_R(X)$).
3. a Nano rg-closed (briefly Nrg-closed) set [15] if Nclo(M) $\subseteq$ T whenever M $\subseteq$ T and T is Nano regular-open in (U, $\tau_R(X)$).
4. a Nano πg-closed (briefly Nπg-closed) set [13] if Nclo(M) $\subseteq$ T whenever M $\subseteq$ T and T is Nano π-open in (U, $\tau_R(X)$).

5. a Nano generalized semi-closed (briefly Ngs-closed) set [4] if Nsclo(M) $\subseteq$ T whenever M $\subseteq$ T and T is Nano open in (U, $\tau_R(X)$).
6. a Nano semi generalized closed (briefly Nsg-closed) set [4] if Nsclo(M) $\subseteq$ T whenever M $\subseteq$ T and T is Nano semi-open in (U, $\tau_R(X)$).
7. an Nano α-generalized closed (briefly Nαg-closed) set [16] if Nαclo(M) $\subseteq$ T whenever M $\subseteq$ T and T is Nano open in (U, $\tau_R(X)$).
8. a Nano generalized pre-closed (briefly Ngp-closed) set [3] Npcl(M) $\subseteq$ T whenever M $\subseteq$ T and T is Nano open in (U, $\tau_R(X)$).
9. a Nano generalized semi pre-closed (briefly Ngsp-closed) set [15] Nspcl(M) $\subseteq$ T whenever M $\subseteq$ T and T is Nano open in (U, $\tau_R(X)$).

The complements of above Nano closed sets is called Nano open sets.

1.7 Remark

The collection of all Ng-closed (resp. Ngs-closed, Nsg-closed, Nαg-closed, N$\hat{g}$-closed, Ngsp-closed, Nano semi-closed, Nano pre-closed, Nano β-closed) sets is denoted by Ngc($\tau_R(X)$) (resp. Ngsc($\tau_R(X)$), Nsgc(($\tau_R(X)$), Nαgc($\tau_R(X)$), N$\hat{g}$c($\tau_R(X)$), Ngspc(($\tau_R(X)$), Nsc(($\tau_R(X)$), Npc(($\tau_R(X)$), Nβc(($\tau_R(X)$)).
We denote the power set of U by P(U).

1.8 Definition

A space (U, $\tau_R(X)$) is called:

(i) T$_{N1/2}$-space [6] if every Ng-closed set is Nano closed.

(ii) T$_{Nb}$-space [6] if every Ngs-closed set is Nano closed.

(iii) NαT$_b$-space [6] if every Nαg-closed set is Nano closed.

(iv) T$_{N\alpha}$-space [7] if every Nano α-closed set in it is Nano closed.

(v) N$_\alpha$T$_d$-space [7] if every Nαg-closed set in it is Ng-closed.

2 N*μ-CLOSED AND N*μ-OPEN SETS

We introduce the definitions

2.1 Definition

A subset M of a space (U, $\tau_R(X)$) is called

1. Nano *g-semi closed set (briefly N*gs-closed) if Nscl(M) $\subseteq$ T whenever M $\subseteq$ T and T is N$\hat{g}$-open in (U, $\tau_R(X)$). The complement of N*gs-closed set is called N*gs-open set.
2. Nano $^*\mu$-closed set (briefly N$^*\mu$-closed) if Ncl(M) $\subseteq$ T whenever M $\subseteq$ T and T is N*gs-open in (U, $\tau_R(X)$). The complement of N$^*\mu$-closed set is called N$^*\mu$-open set.
3. Nano $^*\mu_\alpha$-closed (briefly N$^*\mu_\alpha$-closed) set if Nαclo(M) $\subseteq$ T whenever M $\subseteq$ T and T is N*gs-open in (U, $\tau_R(X)$). The complement of N$^*\mu_\alpha$-closed set is called N$^*\mu_\alpha$-open set.
4. Nano $^*\mu_p$-closed (briefly N$^*\mu_p$-closed) set if Npclo(M) $\subseteq$ T whenever M $\subseteq$ T and T is N*gs-open in (U, $\tau_R(X)$). The complement of N$^*\mu_p$-closed set is called N$^*\mu_p$-open set.

The collection of all N$^*\mu$-closed (resp. N$^*\mu_\alpha$-closed, N$^*\mu_p$-closed) sets is denoted by
N$^*\mu$c(($\tau_R(X)$)(resp. N$^*\mu_\alpha$c(($\tau_R(X)$), N$^*\mu_p$c(($\tau_R(X)$))).

2.2 Proposition

Every Nano closed set is N$^*\mu$-closed.

Proof Let M be a Nano closed set and T be any N*gs-open set containing M. Since M is Nano closed, we have Nclo(M) = M $\subseteq$ T. Hence M is N$^*\mu$-closed. □

The converse of Proposition 2.2 need not be true as seen from the following example.

2.3 Example

Let U = {1, 2, 3, 4} with U/ R= {{3}, {4}, {1, 2}} and X= {2}. The Nano topology $\tau_R(X)$= {ϕ, {1, 2}, U}. Then N$^*\mu$c($\tau_R(X)$) = {ϕ, {3, 4}, {1, 3, 4}, {2, 3, 4}, U}. Here, H = {1, 3, 4} is N$^*\mu$-closed set but not Nano closed.

2.4 Proposition

Every N$^*\mu$-closed set is Ng-closed.

Proof Let M be an N$^*\mu$-closed set and T be any Nano open set containing M. Since every Nano open set is N*gs-open, we have Nclo(M) $\subseteq$ T. Hence M is Ng-closed. □

The converse of Proposition 2.4 need not be true as seen from the following example.

2.5 Example

Let U and $\tau_R(X)$ as in the Example 2.3. Then Ngc($\tau_R(X)$) = {ϕ, {3}, {4}, {1, 3}, {1, 4}, {2, 3}, {2, 4}, {3, 4}, {1, 2, 3}, {1, 2, 4}, {1, 3, 4}, {2, 3, 4}, U}. Here, H = {1, 2, 3} is Ng-closed set but not N$^*\mu$-closed.

2.6 Proposition

Every N*μ-closed set is Nrg-closed.

Proof Let M be an N*μ-closed set and T be any Nano regular open set containing M. Since every Nano regular open set is Nano open set and every Nano open set is N*gs-open set, we have Nclo(M) $\subseteq$ T. Hence M is Nrg-closed. □

The converse of Proposition 2.6 need not be true as seen from the following example.

2.7 Example

Let U = {1, 2, 3, 4} with U/ R= {{1}, {4}, {2, 3}} and X= {1, 3}. The Nano topology $\tau_R(X)$= {ϕ, {1}, {2, 3}, {1, 2, 3}, U}. Then N*μc($\tau_R(X)$) = {ϕ, {4}, {1, 4}, {2, 4}, {3, 4}, {1, 2, 4}, {1, 3, 4}, {2, 3, 4}, U} and Nrgc($\tau_R(X)$) = {ϕ, {4}, {1, 2}, {1, 3}, {1, 4}, {2, 4}, {3, 4}, {1, 2, 3}, {1, 2, 4}, {1, 3, 4}, {2, 3, 4}, U}. Here, H = {1, 2} is Nrg-closed set but not N*μ-closed.

2.8 Proposition

Every N*μ-closed set is Nπg-closed.

Proof Let M be an N*μ-closed set and T be any Nano π-open set containing M. Since every Nano π-open set is Nano open set and every Nano open set is N*gs-open set, we have Nclo(M) $\subseteq$ T. Hence M is Nπg-closed. □

The converse of Proposition 2.8 need not be true as seen from the following example.

2.9 Example

Let U and $\tau_R(X)$ as in the Example 2.3. Then Nπgc($\tau_R(X)$) = $P(U)$. Here, H = {2, 3} is Nπg-closed set but not N*μ-closed.

2.10 Proposition

Every N*μ-closed set is Ngs-closed.

Proof Let M be an N*μ-closed set and T be any Nano open set containing M. Since every Nano open set is N*gs-open, we have Nsclo(M) $\subseteq$ Nclo(M) $\subseteq$ T. Hence M is Ngs-closed. □

The converse of Proposition 2.10 need not be true as seen from the following example.

2.11 Example

Let U and $\tau_R(X)$ as in the Example 2.3. Then Ngsc($\tau_R(X)$) = {ϕ, {3}, {4}, {1, 3}, {1, 4}, {2, 3}, {2, 4}, {3, 4}, {1, 2, 3}, {1, 2, 4}, {1, 3, 4}, {2, 3, 4}, U}. Here, H = {3} is Ngs-closed set but not N*μ-closed.

2.12 Proposition

Every N*μ-closed set is Nsg-closed.

Proof Let M be an N*μ-closed set and T be any Nano semi-open set containing M. Since every Nano semi-open set is N*gs-open, we have Nsclo(M) $\subseteq$ Nclo(M) $\subseteq$ T. Hence M is Ngs-closed. □

The converse of Proposition 2.12 need not be true as seen from the following example.

2.13 Example

Let U and $\tau_R(X)$ as in the Example 2.3. Then Nsgc($\tau_R(X)$) = {ϕ, {3}, {4}, {3, 4}, {1, 3, 4}, {2, 3, 4}, U}. Here, H = {4} is Nsg-closed set but not N*μ-closed.

2.14 Proposition

Every N*μ-closed set is Ngp-closed.

Proof Let M be an N*μ-closed set and T be any Nano open set containing M. Since every Nano open set is N*gs-open, we have Npclo(M)$\subseteq$ Nclo(M) $\subseteq$ T. Hence M is Ngp-closed. □

The converse of Proposition 2.14 need not be true as seen from the following example.

2.15 Example

Let U and $\tau_R(X)$ as in the Example 2.7. Then Ngpc($\tau_R(X)$) = {ϕ, {2}, {3}, {4}, {1, 4}, {2, 4}, {3, 4}, {1, 2, 4}, {1, 3, 4}, {2, 3, 4}, U}. Here, H = {2} is Ngp-closed set but not N*μ-closed.

2.16 Proposition

Every N*μ-closed set is Ngsp-closed.

Proof Let M be an N*μ-closed set and T be any Nano open set containing M. Since every Nano open set is N*g-open, we have Nspclo(M)$\subseteq$ Nclo(M) $\subseteq$ T. Hence M is Ngsp-closed. □

The converse of Proposition 2.16 need not be true as seen from the following example.

2.17 Example

Let U and $\tau_R(X)$ as in the Example 2.3. Then Ngspc($\tau_R(X)$) = {ϕ, {1}, {2}, {3}, {4}, {1, 3}, {1, 4}, {2, 3}, {2, 4}, {3, 4}, {1, 2, 3}, {1, 2, 4}, {1, 3, 4}, {2, 3, 4}, U}. Here, H = {1, 2, 4} is Ngsp-closed set but not N*μ-closed.

2.18 Proposition

Every N*μ-closed set is Nαg-closed.

Proof Let M be an N*μ-closed set and T be any Nano open set containing M. Since every Nano open set is N*gs-open, we have Nαclo(M) $\subseteq$ Nclo(M) $\subseteq$ T. Hence M is Nαg-closed. □

The converse of Proposition 2.18 need not be true as seen from the following example.

2.19 Example

Let U and $\tau_R(X)$ as in the Example 2.3. Then Nαgc($\tau_R(X)$) = {ϕ, {3}, {4}, {1, 3}, {1, 4}, {2, 3}, {2, 4}, {3, 4}, {1, 2, 3}, {1, 2, 4}, {1, 3, 4}, {2, 3, 4}, U}. Here, H = {4} is Nαg-closed set but not N*μ-closed.

2.20 Proposition

Every Nano α-closed set is N*μ_α-closed.

Proof Let M be an Nano α-closed set and T be any N*gs-open set containing M. Since M is Nano α-closed, we have Nαclo(M)= $M$$\subseteq$ T. Hence M is N*μ_α-closed. □

The converse of Proposition 2.20 need not be true as seen from the following example.

2.21 Example

Let U and $\tau_R(X)$ as in the Example 2.3. Then N*μ_αc($\tau_R(X)$) = {ϕ, {3}, {4}, {3, 4}, {1, 3, 4}, {2, 3, 4}, U} and Nαc($\tau_R(X)$) = {ϕ, {3}, {4}, {3, 4}, U}. Here, H = {1, 3, 4} is N*μ_α-closed set but not Nano α-closed.

2.22 Proposition

Every N*μ-closed set is N*μ_α-closed.

Proof Let M be an N*μ-closed set and T be any N*gs-open set containing M. We have Nαclo(M) $\subseteq$ Nclo(M) $\subseteq$ T. Hence M is N*μ_α-closed. □

The converse of Proposition 2.22 need not be true as seen from the following example.

2.23 Example

Let U and $\tau_R(X)$ as in the Example 2.21. Here, H = {3} is N*μ_α-closed but not N*μ-closed.

2.24 Proposition

Every N*μ_α-closed set is N*μ_p-closed.

Proof Let M be an N*μ_α-closed set and T be any N*gs-open set containing M. We have Npclo(M) $\subseteq$ Nαclo(M)$\subseteq$ T. Hence M is N*μ_p-closed. □

The converse of Proposition 2.22 need not be true as seen from the following example.

2.25 Example

Let U and $\tau_R(X)$ as in the Example 2.7. Then N*μ_αc($\tau_R(X)$) = {ϕ, {4}, {1, 4}, {2, 4}, {3, 4}, {1, 2, 4}, {1, 3, 4}, {2, 3, 4}, U} and N*μ_pc($\tau_R(X)$) = {ϕ, {2}, {3}, {4}, {1, 4}, {2, 4}, {3, 4}, {1, 2, 4}, {1, 3, 4}, {2, 3, 4}, U}. Here, H = {3} is N*μ_p-closed set but not N*μ_α-closed.

2.26 Proposition

Every N*μ_α-closed set is Nαg-closed.

Proof Let M be an N*μ_α-closed set and T be any N*gs-open set containing M. We have Nαclo(M)$\subseteq$ T. Hence M is Nαg-closed. □

The converse of Proposition 2.26 need not be true as seen from the following example.

2.27 Example

Let U and $\tau_R(X)$ as in the Example 2.19. Then N*μ_αc($\tau_R(X)$) = {ϕ, {3}, {4}, {3, 4}, {1, 3, 4}, {2, 3, 4}, U}. Here, H = {1, 3} is Nαg-closed set but not N*μ_α-closed.

2.28 Remark

If P and Q are N*μ-closed sets, then P $\cup$ Q is N*μ-closed set.

Proof Let P and Q be any two N*μ-closed sets in (U, $\tau_R(X)$) and G be any N*gs-open set containing P and Q . We have Nclo(P) $\subseteq$ G and Nclo(Q)$\subseteq$G. Thus, Nclo(P $\cup$ Q) = Nclo(P) $\cup$ Nclo(Q) $\subseteq$ G. Hence P $\cup$ Q is N*μ-closed set in (U, $\tau_R(X)$). □

2.29 Remark

If K and L are N*μ-closed sets, then K $\cap$ L is a N*μ-closed set.

2.30 Example

Let U and $\tau_R(X)$ as in the Example 2.7. Here, k = {1, 4} and L = {2, 4} are N*μ-closed sets but K $\cap$ L = {1, 2, 4} is a N*μ-closed set.

2.31 Proposition

If A subset M of (U, $\tau_R(X)$) is a N*μ-closed if and only if Nclo(A) − M does not contain any nonempty N*gs-closed set.

Proof Necessity. Suppose that M is N*μ-closed. Let S be a N*gs-closed subset of Nclo(M) − M. Then M ⊆ S^c . Since M is N*μ-closed, we have Nclo(M) ⊆ S^c . Consequently, S ⊆ $(Nclo(M))^c$. Hence, S ⊆ Nclo(M) ∩ $(Nclo(M))^c$ $= \phi$. Therefore S is empty.

Sufficiency. Suppose that Nclo(M) − M contains no nonempty N*gs-closed set. Let M ⊆ G and G be N*gs-closed If Nclo(M) ≠ G, then Nclo(M) ⊆ G^c $\neq \phi$. Since Nclo(M) is a Nano closed set and G^c is a N*gs-closed set, Nclo(M) ∩ G^c is a nonempty N*gs-closed subset of Nclo(M) − M. This is a contradiction. Therefore, Nclo(M) ⊆ G and hence M is N*μ-closed. □

2.32 Proposition

If A is N*μ-closed in (U, $\tau_R(X)$) such that A ⊆ B ⊆ Nclo(A), then B is also a N*μ-closed set of (U, $\tau_R(X)$).

Proof Let W be a N*gs-open set of (U, $\tau_R(X)$) such that B ⊆ W. Then A ⊆ W. Since A is N*μ-closed, we get, Nclo(A) ⊆ W. Now Nclo(B) ⊆ Nclo(Nclo(A)) = Nclo(A) ⊆ W. Therefore, B is also a N*μ-closed set of (U, $\tau_R(X)$). □

2.33 Definition

The intersection of all N*gs-open subsets of (U, $\tau_R(X)$) containing A is called the Nano *gs-kernel of A and denoted by N*gs-ker(A).

2.34 Lemma

A subset A of (U, $\tau_R(X)$) is N*μ-closed if and only if Nclo(A) ⊆ N*gs-ker(A).

Proof Suppose that A is N*μ-closed. Then Nclo(A) ⊆ U whenever A ⊆ U and U is N*gs-open. Let x ∈ Nclo(A). If x ∉ N*gs-ker(A), then there is a N*gs-open set U containing A such that x ∉ U. Since U is a N*gs-open set containing A, we have x ∉ Nclo(A) and this is a contradiction.

Conversely, let Nclo(A) ⊆ N*gs-ker(A). If U is any N*gs-open set containing A, then Nclo(A) ⊆ N*gs-ker(A) ⊆ U. Therefore, A is N*μ-closed. □

2.35 Definition

A subset M of a space U is said to be N*μ-open if M^C is N*μ-closed.

The class of all N*μ-open subsets of U is denoted by N*μo($\tau_R(X)$).

2.36 Proposition

1. Every Nano open set is N*μ-open set but not conversely.
2. Every N*μ-open set is Ng-open set but not conversely.
3. Every N*μ-open set is Nrg-open set but not conversely.

4. Every N*μ-open set is Nπg-open set but not conversely.
5. Every N*μ-open set is Ngs-open set but not conversely.
6. Every N*μ-open set is Nsg-open set but not conversely.
7. Every N*μ-open set is Ngp-open set but not conversely.
8. Every N*μ-open set is Ngsp-open set but not conversely.
9. Every N*μ-open set is Nαg-open set but not conversely.
10. Every Nano α-open set is N*μ_α-open but not conversely.
11. Every N*μ-open set is N*μ_α-open set but not conversely.
12. Every N*μ_α-open set is N*μ_p-open but not conversely.

Proof Omitted. □

2.37 Proposition

A subset M of a Nano topological space U is said to N*μ-open if and only if P $\subseteq$ Ninto(M) whenever M $\supseteq$ P and P is N*gs-closed in U.

Proof Suppose that M is N*μ-open in U and M $\supseteq$ P, where P is N*gs-closed in U. Then $M^c \subseteq P^c$, where P^c is N*gs-open-open in U. Hence we get Nclo (M^c) $\subseteq$ P^c implies ($Ninto(M))^c \subseteq P^c$. Thus, we have Ninto(M) $\supseteq$ P. conversely, suppose that M^c $\subseteq$ T and T is N*gs-open-open in U then M $\supseteq$ T^c and T^c is N*gs-closed then by hypothesis Ninto(M) $\supseteq$ T^c implies ($Ninto(M))^c$ $\subseteq$ T. Hence Nclo (M^c) $\subseteq$ T gives M^c is N*μ-closed. □

2.38 Proposition

In a Nano topological space U, for each u $\in$ U, either {u} is N*gs-closed or N*μ-open in U.

Proof Suppose that {u} is not N*gs-closed in U. Then $\{u\}^c$ is not N*gs-open-open and the only N*gs-open set containing $\{u\}^C$ is the space U itself. Therefore, Nclo ($\{u\}^C$) $\subseteq$ U and so $\{u\}^C$ is N*μ-closed gives {u} is N*μ-open. □

3 T$_{N^*\mu}$-SPACES

We introduce the following definition.

3.1 Definition

A space (U, $\tau_R(X)$) is called a T$_{N^*\mu}$-space if every N*μ-closed set in it is Nano closed.

3.2 Example

Let U = {1, 2, 3}, with U/ R= {{1}, {2, 3}} and X= {1}. Then the Nano topology $\tau_R(X)$ = {ϕ, {1}, U}. Here N*μc($\tau_R(X)$) = {ϕ, {2, 3}, U}. Thus (U, $\tau_R(X)$) is a $T_{N^*\mu}$-space.

3.3 Example

Let U and $\tau_R(X)$ as in the Example 2.7. Then N*μc($\tau_R(X)$) = {ϕ, {q}, {p, q}, {q, r}, U}. Thus (U, $\tau_R(X)$) is not a $T_{N^*\mu}$-space.

3.4 Proposition

Every $T_{N1/2}$-space is $T_{N^*\mu}$-space but not conversely.

Proof Follows from Proposition 2.4. □

The converse of Proposition 3.4 need not be true as seen from the following example.

3.5 Example

Let U and $\tau_R(X)$ as in the Example 3.2. Ngc($\tau_R(X)$) = {ϕ, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, U}. Thus (U, $\tau_R(X)$) is not a $T_{N1/2}$-space.

3.6 Proposition

Every $N_\alpha T_b$-space is $T_{N^*\mu}$-space but not conversely.

Proof Follows from Proposition 2.18. □

The converse of Proposition 3.6 need not be true as seen from the following example.

3.7 Example

Let U and $\tau_R(X)$ as in the Example 3.2. Nαgc($\tau_R(X)$) = {ϕ, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, U}. Thus (U, $\tau_R(X)$) is not a $N_\alpha T_b$-space.

3.8 Proposition

Every T_{Nb}-space is $T_{N^*\mu}$-space but not conversely.

Proof Follows from Proposition 2.10. □

The converse of Proposition 3.8 need not be true as seen from the following example.

3.9 Example

Let U and $\tau_R(X)$ as in the Example 3.2. Ngsc($\tau_R(X)$) = {ϕ, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, U}. Thus (U, $\tau_R(X)$) is not a T_{Nb}-space.

3.10 Remark

$T_{N^*\mu}$-spaces and $T_{N\alpha}$-spaces are independent.

3.11 Example

Let U and $\tau_R(X)$ as in the Example 3.2, Nαc($\tau_R(X)$) = {ϕ, {2}, {3}, {2, 3}, U}. Thus (U, $\tau_R(X)$) is a $T_{N^*\mu}$-space but not a $T_{N\alpha}$-space.

3.12 Example

Let U = {1, 2, 3}, with U/ R= {{3}, {1, 2}, {2, 1}} and X= {1, 2}. Then the Nano topology $\tau_R(X)$ = {ϕ, {1, 2}, U}. Then N*μc($\tau_R(X)$) = {ϕ, {3}, {1, 3}, {2, 3}, U} and Nαc($\tau_R(X)$) = {ϕ, {3}, U}. Thus (U, $\tau_R(X)$) is a $T_{N\alpha}$-space but not $T_{N^*\mu}$-space.

3.13 Theorem

For a space (U, $\tau_R(X)$) the following properties are equivalent:

(i) (U, $\tau_R(X)$) is a $T_{N^*\mu}$-space.

(ii) Every singleton subset of (U, $\tau_R(X)$) is either N*gs-closed or Nano open.

Proof (i) $\rightarrow$ (ii). Assume that for some u $\in$ U, the set {u} is not a N*gs-closed in (U, $\tau_R(X)$). Then the only N*gs-open-open set containing $\{u\}^c$ is U and so $\{u\}^c$ is N*μ-closed in (U, $\tau_R(X)$). By assumption $\{u\}^c$ is Nano closed in (U, $\tau_R(X)$) or equivalently {u} is Nano open.

(ii) $\rightarrow$ (i). Let M be a N*μ-closed subset of (U, $\tau_R(X)$) and let u $\in$ Nclo(M). By assumption {u} is either N*gs-closed or Nano open.

Case (a) Suppose that {u} is N*gs-closed. If u $\notin$ M, then Nclo(M) - M contains a nonempty N*gs-closed set {u}, which is a contradiction to Theorem 2.31. Therefore x $\in$ M.

Case (b) Suppose that {u} is Nano open. Since u $\in$ Nclo(M), {u} $\cap$ M $\neq \phi$ and so u $\in$ M. Thus in both case, u $\in$ M and therefore Nclo(M) $\subseteq$ M or equivalently M is a Nano closed set of (U, $\tau_R(X)$). □

4 ${}_g$T$_{N^*\mu}$-SPACES

4.1 Definition

A space (U, $\tau_R(X)$) is called a ${}_g$T$_{N^*\mu}$-space if every Ng-closed set in it is N*μ-closed.

4.2 Example

Let X and τ as in the Example 2.7, is a ${}_g$T$_{N^*\mu}$-space and the space (U, $\tau_R(X)$) in the Example 3.2, is not a ${}_g$T$_{N^*\mu}$-space.

4.3 Proposition

Every T$_{N1/2}$-space is ${}_g$T$_{N^*\mu}$-space but not conversely.

Proof Follows from Proposition 2.2. □

The converse of Proposition 4.3 need not be true as seen from the following example.

4.4 Example

Let X and τ as in the Example 2.7, is a ${}_g$T$_{N^*\mu}$-space but not a T$_{N1/2}$-space.

4.5 Remark

T$_{N^*\mu}$-space and ${}_g$T$_{N^*\mu}$-space are independent.

4.6 Example

The space (U, $\tau_R(X)$) in the Example 2.7, is a ${}_g$T$_{N^*\mu}$-space but not a T$_{N^*\mu}$-space and the space (U, $\tau_R(X)$) in the Example 3.2, is a T$_{N^*\mu}$-space but not a ${}_g$T$_{N^*\mu}$-space.

4.7 Theorem

If (U, $\tau_R(X)$) is a ${}_g$T$_{N^*\mu}$-space, then every singleton subset of (U, $\tau_R(X)$) is either Ng-closed or N*μ-open.

Proof Assume that for some x $\in$ X, the set {x} is not a Ng-closed in (U, $\tau_R(X)$). Then {x} is not a Nano closed set, since every Nano closed set is a Ng-closed set. So $\{x\}^c$ is not Nano open and the only Nano open set containing $\{x\}^c$ is X itself. Therefore $\{x\}^c$ is trivially a Ng-closed set and by assumption, $\{x\}^c$ is an N*μ-closed set or equivalently {x} is N*μ-open. □

The converse of Theorem 4.7 need not be true as seen from the following example.

4.8 Example

Let X and τ as in the Example 3.2. The sets {2} and {3} are Ng-closed in (U, $\tau_R(X)$) and the set {1} is N*μ-open. But the space (U, $\tau_R(X)$) is not a $_g$T$_{N^*\mu}$-space.

4.9 Theorem

A space (U, $\tau_R(X)$) is T$_{N1/2}$ if and only if it is both T$_{N^*\mu}$ and $_g$T$_{N^*\mu}$.

Proof Necessity. Follows from Propositions 3.4 and 4.3.
Sufficiency. Assume that (U, $\tau_R(X)$) is both T$_{N^*\mu}$ and $_g$T$_{N^*\mu}$. Let A be a Ng-closed set of (U, $\tau_R(X)$). Then A is N*μ-closed, since (U, $\tau_R(X)$) is a $_g$T$_{N^*\mu}$. Again since (U, $\tau_R(X)$) is a T$_{N^*\mu}$, A is a Nano closed set in (U, $\tau_R(X)$) and so (U, $\tau_R(X)$) is a T$_{1/2}$. □

5 $_\alpha$T$_{N^*\mu}$-SPACES

5.1 Definition

A space (U, $\tau_R(X)$) is called a $_\alpha$T$_{N^*\mu}$-space if every Nαg-closed set in it is N*μ-closed.

5.2 Example

Let U = {1, 2, 3}, with U/ R= {{1}, {2, 3}} and X= {1, 3}. Then the Nano topology $\tau_R(X)$ = {ϕ, {1}, {2, 3}, U}. Thus (U, $\tau_R(X)$) is a $_\alpha$T$_{N^*\mu}$-space and the space (U, $\tau_R(X)$) in the Example 2.3, is not a $_\alpha$T$_{N^*\mu}$-space.

5.3 Proposition

Every N$_\alpha$T$_b$-space is $_\alpha$T$_{N^*\mu}$-space but not conversely.

Proof Follows from Proposition 2.2. □

The converse of Proposition 6.3 need not be true as seen from the following example.

5.4 Example

Let X and $\tau_R(X)$ in the Example 5.2, is a $_\alpha$T$_{N^*\mu}$-space but not a N$_\alpha$T$_b$-space.

5.5 Proposition

Every $_\alpha$T$_{N^*\mu}$-space is a N$_\alpha$T$_d$-space but not conversely.

Proof Let (U, $\tau_R(X)$) be an $_\alpha$T$_{N^*\mu}$-space and let A be an Nαg-closed set of (U, $\tau_R(X)$). Then A is a N*μ-closed subset of (U, $\tau_R(X)$) and by Proposition 2.4, A is Ng-closed. Therefore (U, $\tau_R(X)$) is an N$_\alpha$T$_d$-space. □

The converse of Proposition 5.5 need not be true as seen from the following example.

5.6 Example

Let X and $\tau_R(X)$ in the Example 2.3, is a $N_\alpha T_d$-space but not a $_\alpha T_{N^*\mu}$-space.

5.7 Theorem

If (U, $\tau_R(X)$) is a $_\alpha T_{N^*\mu}$-space, then every singleton subset of (U, $\tau_R(X)$) is either Nαg-closed or N$^*\mu$-open.

Proof Similar to Theorem 4.7. □

The converse of Theorem 5.7 need not be true as seen from the following example.

5.8 Example

Let X and $\tau_R(X)$ as in the Example 3.2. The sets {2} and {3} are Nαg-closed in (U, $\tau_R(X)$) and the set {1} is N$^*\mu$-open. But the space (U, $\tau_R(X)$) is not a $_\alpha T_{N^*\mu}$-space.

Conclusions In this paper is to introduce a new class of sets called N$^*\mu$-closed sets in Nano topological spaces and to study some of its basic properties. As applications of N$^*\mu$-closed sets, we introduce $T_{N^*\mu}$-spaces, $_gT_{N^*\mu}$-spaces and $_\alpha T_{N^*\mu}$-spaces. Moreover, we obtain certain new characterizations for the $T_{N^*\mu}$-spaces, $_gT_{N^*\mu}$-spaces and $_\alpha T_{N^*\mu}$-spaces. In future, we have extend this work in various nano topological field with some applications.

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