On $ng\mu$-closed sets

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Abstract

The aim of this paper, we offer a new class of sets called $ng\mu$-closed sets in nano topological spaces and we study some of its basic properties. We introduce and study $ng\mu$-continuous, $ng\mu$-irresolute and contra $ng\mu$-continuous. Moreover, we obtain their properties and characterizations.

2020 Mathematics Subject Classification: 54C10, 54A05, Secondary 54D15, 54D30.

Keywords: $ng\mu$-closed sets, $ng\mu$-open sets, $T_{ng\mu}$-spaces, $ng\mu$-continuous $ng\mu$-irresolute and contra $ng\mu$-continuous.

1 Introduction

Bhuvaneswari and Mythili Gnanapriya [1] introduced and studied nano generalised closed sets. S. Ganesan et al [4] introduced and studied n*g-closed sets. In this paper, we introduce and study some basic properties of $ng\mu$-closed sets and $ng\mu$-open sets. We introduce and study $ng\mu$-continuous, $ng\mu$-irresolute and contra $ng\mu$-continuous. Moreover, we obtain their properties and characterizations.

2 Preliminaries

2.1 Definition

[7] If (K, $\tau_R(X)$) is the nano topological space with respect to X where X $\subseteq$ K and if M $\subseteq$ K, then

1. The nano interior of the set M is defined as the union of all nano open subsets contained in M and it is denoted by ninte(M). That is, ninte(M) is the largest nano open subset of M.

2. The nano closure of the set M is defined as the intersection of all nano closed sets containing M and it is denoted by nclo(M). That is, nclo(M) is the smallest nano closed set containing M.

2.2 Definition

A subset M of a space (K, $\tau_R(X)$) is called:

1. nano α-open set [7] if M $\subseteq$ ninte(nclo(ninte(M))).
2. nano pre-open set [7] if M $\subseteq$ ninte(nclo(M)).

The complements of the above mentioned nano open sets are called their respective nano closed sets.
The nano α-closure [5] (resp. nano pre-closure [3]) of a subset M of U, denoted by nαclo(M) (resp. npclo(M)) is defined to be the intersection of all nano α-closed (resp. nano pre closed) sets of (K, $\tau_R(X)$) containing M.

2.3 Definition

A subset M of a space (K, $\tau_R(X)$) is called

1. ng-closed set [1] if nclo(M) $\subseteq$ T whenever M $\subseteq$ T and T is nano open in (K, $\tau_R(X)$).
2. an nαg-closed set [10] if nαclo(M) $\subseteq$ T whenever M $\subseteq$ T and T is nano open in (K, $\tau_R(X)$).

The complements of above nano closed sets is called nano open sets.

2.4 Definition

A map f : (K, $\tau_R(X)$) $\to$ (L, $\tau'_R(Y)$) is called:

1. nano continuous [8] if f^{-1}(W) is a nano closed set of K for every nano closed set W of L.
2. nano α-continuous [9] if f^{-1}(W) is an nano α-closed set in K for every nano closed set W of L.
3. ng-continuous [2] if f^{-1}(W) is a ng-closed set of K for every nano closed set W of L.
4. nαg-continuous [11] if f^{-1}(W) is a nαg-closed set of K for every nano closed set W of L.

2.5 Definition

[6] A map f : (K, $\tau_R(X)$) $\to$ (L, $\tau'_R(Y)$) is called contra nano continuous if f^{-1}(W) is a nano closed set of (K, $\tau_R(X)$) for every nano open set W of (L, $\tau'_R(Y)$).

2.6 Definition

[12] A function f: (O, $\mathcal{N}$) $\to$ (P, $\mathcal{N}'$) is said to be nano contra g-continuous if f^{-1}(V) is a ng-closed set of (O, $\mathcal{N}$) for every n-open set V of (P, $\mathcal{N}'$).

3 $ng\mu$-Closed and $ng\mu$-Open sets

We introduce the definitions

3.1 Definition

A subset M of a space (K, $\tau_R(X)$) is called

1. a ngα*-closed set if nαclo(M) $\subseteq$ ninte(T) whenever M $\subseteq$ T and T is nα-open in (K, $\tau_R(X)$). The complement of ngα*-closed set is called ngα*-open set.
2. a nμ-closed set if nclo(M) $\subseteq$ T whenever M $\subseteq$ T and T is ngα*-open in (K, $\tau_R(X)$). The complement of nμ-closed set is called nμ-open set.
3. a generalized nμ-closed (briefly $ng\mu$-closed) set if nclo(M) $\subseteq$ T whenever M $\subseteq$ T and T is nμ-open in (K, $\tau_R(X)$). The complement of $ng\mu$-closed set is called $ng\mu$-open set.

3.2 Proposition

Every nano closed set is $ng\mu$-closed.
Proof Let M be a nano closed set and T be any nμ-open set containing M. Since M is nano closed, we have nclo(M) = M $\subseteq$ T. Hence M is $ng\mu$-closed. □

3.3 Example

Let K = {1, 2, 3, 4} with K/ R= {{3}, {4}, {1, 2}} and X= {2}. The nano topology $\tau_R(X)$= {ϕ, {1, 2}, K}. Then $ng\mu$-closed sets are ϕ, {3, 4}, {1, 3, 4}, {2, 3, 4}, K. Here, H = {1, 3, 4} is $ng\mu$-closed set but not nano closed.

3.4 Proposition

Every $ng\mu$-closed set is ng-closed.
Proof Let M be an $ng\mu$-closed set and T be any nano open set containing M. Since every Nano open set is nμ-open, we have nclo(M) $\subseteq$ T. Hence M is ng-closed. □

3.5 Example

Let K and $\tau_R(X)$ as in the Example 3.3. Then ng-closed sets are ϕ, {3}, {4}, {1, 3}, {1, 4}, {2, 3}, {2, 4}, {3, 4}, {1, 2, 3}, {1, 2, 4}, {1, 3, 4}, {2, 3, 4}, K. Here, H = {2, 3} is ng-closed set but not $ng\mu$-closed.

3.6 Definition

A subset M of a space (K, $\tau_R(X)$) is called

1. nano gμ_α-closed (briefly $ng\mu_\alpha$-closed) set if nαclo(M) $\subseteq$ T whenever M $\subseteq$ T and T is nμ-open in (K, $\tau_R(X)$). The complement of $ng\mu_\alpha$-closed set is called $ng\mu_\alpha$-open set.
2. nano gμ_p-closed (briefly $ng\mu_p$-closed) set if npclo(M) $\subseteq$ T whenever M $\subseteq$ T and T is nμ-open in (K, $\tau_R(X)$). The complement of $ng\mu_p$-closed set is called $ng\mu_p$-open set.

3.7 Proposition

Every nano α-closed set is $ng\mu_\alpha$-closed.

Proof Let M be an nano α-closed set and T be any nμ-open set containing M. Since M is nano α-closed, we have nαclo(M)$= M\subseteq$ T. Hence M is $ng\mu_\alpha$-closed. □

3.8 Example

Let K and $\tau_R(X)$ as in the Example 3.3. Then $ng\mu_\alpha$-closed sets are ϕ, {3}, {4}, {3, 4}, {1, 3, 4}, {2, 3, 4}, K and nα-closed sets are ϕ, {3}, {4}, {3, 4}, K. Here, H = {2, 3, 4} is $ng\mu_\alpha$-closed set but not nano α-closed.

3.9 Proposition

Every $ng\mu$-closed set is $ng\mu_\alpha$-closed.

Proof Let M be an $ng\mu$-closed set and T be any nμ-open set containing M. We have nαclo(M) $\subseteq$ nclo(M) $\subseteq$ T. Hence M is $ng\mu_\alpha$-closed. □

3.10 Example

Let K and $\tau_R(X)$ as in the Example 3.8. Here, H = {4} is $ng\mu_\alpha$-closed but not $ng\mu$-closed.

3.11 Proposition

Every $ng\mu_\alpha$-closed set is $ng\mu_p$-closed.

Proof Let M be an $ng\mu_\alpha$-closed set and T be any nμ-open set containing M. We have npclo(M) $\subseteq$ nαclo(M)$\subseteq$ T. Hence M is $ng\mu_p$-closed. □

3.12 Example

Let K and $\tau_R(X)$ as in the Example 3.8. Then $ng\mu_p$-closed sets are ϕ, {1}, {2}, {3}, {4}, {1, 3}, {1, 4}, {2, 3}, {2, 4}, {3, 4}, {1, 3, 4}, {2, 3, 4}, K. Here, H = {1} is $ng\mu_p$-closed set but not $ng\mu_\alpha$-closed.

3.13 Proposition

Every $ng\mu_\alpha$-closed set is nαg-closed.

Proof Let M be an $ng\mu_\alpha$-closed set and T be any nano open set containing M. Since every nano open set is nμ-open, we have nαclo(M)$\subseteq$ nclo(M) $\subseteq$ T. Hence M is nαg-closed. □

3.14 Example

Let K and $\tau_R(X)$ as in the Example 3.8. Then nαg-closed sets are ϕ, {3}, {4}, {1, 3}, {1, 4}, {2, 3}, {2, 4}, {3, 4}, {1, 2, 3}, {1, 2, 4}, {1, 3, 4}, {2, 3, 4}, K. Here, H = {1, 4} is nαg-closed set but not $ng\mu_\alpha$-closed.

3.15 Remark

If S and G are $ng\mu$-closed sets, then S $\cup$ G is $ng\mu$-closed set. **Proof** Let S and G be any two $ng\mu$-closed sets in (K, $\tau_R(X)$) and T be any nμ-open set containing S and G . We have nclo(S) $\subseteq$ T and nclo(G)$\subseteq$ T. Thus, nclo(S $\cup$ G) = nclo(S) $\cup$ nclo(G) $\subseteq$ T. Hence S $\cup$ G is $ng\mu$-closed set in (K, $\tau_R(X)$). □

3.16 Remark

If S and G are $ng\mu$-closed sets, then S $\cap$ G is a $ng\mu$-closed set.

3.17 Example

Let K and $\tau_R(X)$ as in the Example 3.3. Here, S = {1, 3, 4} and G = {2, 3, 4} are $ng\mu$-closed sets but S $\cap$ G = {3, 4} is a $ng\mu$-closed set.

3.18 Proposition

If A subset M of (K, $\tau_R(X)$) is a $ng\mu$-closed if and only if nclo(M) $-$ M does not contain any nonempty nμ-closed set.

Proof Necessity. Suppose that M is $ng\mu$-closed. Let S be a nμ-closed subset of nclo(M) $-$ M. Then M $\subseteq$ S^c . Since M is $ng\mu$-closed, we have nclo(M) $\subseteq$ S^c . Consequently, S $\subseteq$ $(nclo(M))^c$. Hence, S $\subseteq$ nclo(M) $\cap$ $(nclo(M))^c$ $= \phi$. Therefore S is empty.

Sufficiency. Suppose that nclo(M) $-$ M contains no nonempty nμ-closed set. Let M $\subseteq$ G and G be nμ-closed If nclo(M) $\neq$ G, then nclo(M) $\subseteq$ G^c $\neq \phi$. Since nclo(M) is a nano closed set and G^c is a nμ-closed set, nclo(M) $\cap$ G^c is a nonempty nμ-closed subset of nclo(M) $-$ M. This is a contradiction. Therefore, nclo(M) $\subseteq$ G and hence M is $ng\mu$-closed. □

3.19 Proposition

If A is $ng\mu$-closed in (K, $\tau_R(X)$) such that A $\subseteq$ B $\subseteq$ nclo(A), then B is also a $ng\mu$-closed set of (K, $\tau_R(X)$).

Proof Let W be a nμ-open set of (K, $\tau_R(X)$) such that B $\subseteq$ W. Then A $\subseteq$ W. Since A is $ng\mu$-closed, we get, nclo(A)

$\subseteq$ W. Now nclo(B) $\subseteq$ nclo(nclo(A)) = nclo(A) $\subseteq$ W. Therefore, B is also a $ng\mu$-closed set of (K, $\tau_R(X)$). □

3.20 Definition

The intersection of all nμ-open subsets of (K, $\tau_R(X)$) containing M is called the nano μ-kernel of M and denoted by nμ-ker(M).

3.21 Lemma

A subset M of (K, $\tau_R(X)$) is $ng\mu$-closed if and only if nclo(M) $\subseteq$ nμ-ker(M).

Proof Suppose that M is $ng\mu$-closed. Then nclo(M) $\subseteq$ T whenever M $\subseteq$ T and T is nμ-open. Let k $\in$ nclo(M). If k $\notin$ nμ-ker(M), then there is a nμ-open set T containing M such that k $\notin$ T. Since T is a nμ-open set containing M, we have k $\notin$ nclo(M) and this is a contradiction. Conversely, let nclo(M) $\subseteq$ nμ-ker(M). If T is any nμ-open set containing M, then nclo(M) $\subseteq$ nμ-ker(M) $\subseteq$ T. Therefore, M is $ng\mu$-closed. □

3.22 Definition

A subset M of a space K is said to be $ng\mu$-open if M^C is $ng\mu$-closed.

3.23 Proposition

1. Every nano open set is $ng\mu$-open set but not conversely.
2. Every $ng\mu$-open set is ng-open set but not conversely.
3. Every nano α-open set is $ng\mu_\alpha$-open but not conversely.
4. Every $ng\mu$-open set is $ng\mu_\alpha$-open set but not conversely.
5. Every $ng\mu_\alpha$-open set is $ng\mu_p$-open but not conversely.
6. Every $ng\mu_\alpha$-open set is nαg-open but not conversely.

Proof Omitted. □

3.24 Proposition

A subset M of a nano topological space K is said to $ng\mu$-open if and only if P $\subseteq$ ninto(M) whenever M $\supseteq$ P and P is nμ-closed in U.

Proof Suppose that M is $ng\mu$-open in K and M $\supseteq$ P, where P is nμ-closed in K. Then $M^c \subseteq P^c$, where P^c is nμ-open-open in K. Hence we get nclo (M^c) $\subseteq$ P^c implies ($ninte(M))^c \subseteq P^c$. Thus, we have ninte(M) $\supseteq$ P. conversely, suppose that M^c $\subseteq$ T and T is nμ-open in K then M $\supseteq$ T^c and T^c is nμ-closed then by hypothesis ninte(M) $\supseteq$ T^c implies ($ninte(M))^c$ $\subseteq$ T. Hence nclo (M^c) $\subseteq$ T gives M^c is $ng\mu$-closed. □

3.25 Proposition

In a nano topological space U, for each u ∈ U, either {u} is nμ-closed or $ng\mu$-open in U.

Proof Suppose that {u} is not nμ-closed in U. Then $\{u\}^c$ is not nμ-open and the only nμ-open set containing $\{u\}^C$ is the space U itself. Therefore, nclo ($\{u\}^C$) ⊆ U and so $\{u\}^C$ is $ng\mu$-closed gives {u} is $ng\mu$-open. □

4 $ng\mu$-Continuous maps and Irresolute maps

We introduce the following definition.

4.1 Definition

A map f : (K, $\tau_R(X)$) → (L, $\tau'_R(Y)$) is called $ng\mu$-continuous if f^{-1}(W) is a $ng\mu$-closed set of (K, $\tau_R(X)$) for every nano closed set W of (L, $\tau'_R(Y)$).

4.2 Proposition

1. Every nano continuous is $ng\mu$-continuous but not conversely.
2. Every $ng\mu$-continuous is ng-continuous but not conversely.

Proof Omitted. □

4.3 Definition

A map f : (K, $\tau_R(X)$) → (L, $\tau'_R(Y)$) is called $ng\mu_\alpha$-continuous (resp. $ng\mu_p$-continuous) if f^{-1}(W) is a $ng\mu_\alpha$-closed (resp. $ng\mu_p$-closed) set of (K, $\tau_R(X)$) for every nano closed set W of (L, $\tau'_R(Y)$).

4.4 Proposition

1. Every nano α-continuous is $ng\mu_\alpha$-continuous but not conversely.
2. Every $ng\mu$-continuous is $ng\mu_\alpha$-continuous but not conversely.
3. Every $ng\mu_\alpha$-continuous is $ng\mu_p$-continuous but not conversely.
4. Every $ng\mu_\alpha$-continuous is nαg-continuous but not conversely.

Proof Omitted. □

4.5 Theorem

If f : (K, $\tau_R(X)$) → (L, $\tau'_R(Y)$) is $ng\mu$-continuous and g : (L, $\tau'_R(Y)$) → (M, $\tau^*_R(Z)$) is nano continuous then g ∘ f : (K, $\tau_R(X)$) → (M, $\tau^*_R(Z)$) is $ng\mu$-continuous.

Proof Let G be nano closed set in M. Since g is nano continuous, g^{-1}(G) is nano closed in L. Since f is $ng\mu$-continuous, $(g \circ f)^{-1}$(G) = $f^{-1}(g^{-1}$(G)) is $ng\mu$-closed in K. Therefore g ∘ f is $ng\mu$-continuous. □

4.6 Proposition

A map f : (K, $\tau_R(X)$) → (L, $\tau'_R(Y)$) is $ng\mu$-continuous if and only if f^{-1}(W) is $ng\mu$-open in (K, $\tau_R(X)$) for every nano open set W in (L, $\tau'_R(Y)$).

Proof Let f : (K, $\tau_R(X)$) → (L, $\tau'_R(Y)$) be $ng\mu$-continuous and W be an nano open set in (L, $\tau'_R(Y)$). Then W^c is nano closed in (L, $\tau'_R(Y)$) and since f is $ng\mu$-continuous, $f^{-1}(W^c)$ is $ng\mu$-closed in (K, $\tau_R(X)$). But $f^{-1}(W^c) = f^{-1}((W))^c$ and so f^{-1}(W) is $ng\mu$-open in (K, $\tau_R(X)$).

Conversely, assume that f^{-1}(W) is $ng\mu$-open in (K, $\tau_R(X)$) for each nano open set W in (L, $\tau'_R(Y)$). Let F be a nano closed set in (L, $\tau'_R(Y)$). Then F^c is nano open in (L, $\tau'_R(Y)$) and by assumption, $f^{-1}(F^c)$ is $ng\mu$-open in (K, $\tau_R(X)$). Since $f^{-1}(F^c) = f^{-1}((F))^c$, we have f^{-1}(F) is nano closed in (K, $\tau_R(X)$)and so f is $ng\mu$-continuous. □

4.7 Definition

A space (K, $\tau_R(X)$) is called a $T_{ng\mu}$-space if every $ng\mu$-closed set in it is nano closed.

4.8 Theorem

Let f : (K, $\tau_R(X)$) → (L, $\tau'_R(Y)$) be an $ng\mu$-continuous map. If (K, $\tau_R(X)$), the domain of f is an $T_{ng\mu}$-space, then f is nano continuous.

Proof Let W be a nano closed set of (L, $\tau'_R(Y)$). Then f^{-1}(W) is a $ng\mu$-closed set of (K, $\tau_R(X)$), since f is $ng\mu$-continuous. Since (K, $\tau_R(X)$) is an $T_{ng\mu}$-space, then f^{-1}(W) is a nano closed set of (K, $\tau_R(X)$). Therefore f is nano continuous. □

4.9 Definition

A map f : (K, $\tau_R(X)$) → (L, $\tau'_R(Y)$) is called $ng\mu$-irresolute if f^{-1}(W) is a $ng\mu$-closed set of (K, $\tau_R(X)$) for every $ng\mu$-closed set W of (L, $\tau'_R(Y)$).

4.10 Theorem

Every $ng\mu$-irresolute map is $ng\mu$-continuous but not conversely.

Proof Let f : (K, $\tau_R(X)$) → (L, $\tau'_R(Y)$)be a $ng\mu$-irresolute map. Let W be a nano closed set of (L, $\tau'_R(Y)$). Then by the Proposition 3.2, W is $ng\mu$-closed. Since f is $ng\mu$-irresolute, then f^{-1}(W) is a $ng\mu$-closed set of (K, $\tau_R(X)$). Therefore f is $ng\mu$-continuous. □

4.11 Theorem

Let f : (K, $\tau_R(X)$) → (L, $\tau'_R(Y)$) and g : (L, $\tau'_R(Y)$) → (M, $\tau^*_R(Z)$) be any two maps. Then

1. g ∘ f is $ng\mu$-continuous if g is nano continuous and f is $ng\mu$-continuous.
2. g ∘ f is $ng\mu$-irresolute if both f and g are $ng\mu$-irresolute.
3. g ∘ f is $ng\mu$-continuous if g is $ng\mu$-continuous and f is $ng\mu$-irresolute.

Proof Omitted. □

4.12 Definition

A map f : (K, $\tau_R(X)$) → (L, $\tau'_R(Y)$) is called contra $ng\mu$-continuous if f^{-1}(W) is a $ng\mu$-closed set of (K, $\tau_R(X)$) for every nano open set W of (L, $\tau'_R(Y)$).

4.13 Proposition

Every contra nano continuous is contra $ng\mu$-continuous but not conversely.

Proof Let f : (K, $\tau_R(X)$) → (L, $\tau'_R(Y)$) be a contra nano continuous map and let G be any nano open set in (L, $\tau'_R(Y)$). Then, f^{-1}(G) is nano closed in K. Since every nano closed set is $ng\mu$-closed, f^{-1}(G) is $ng\mu$-closed in K. Therefore f is contra $ng\mu$-continuous. □

4.14 Example

Let K = {1, 2, 3} with K/R = {{2}, {1, 3}, {3, 1}} and X = {1, 3}. Then nano topology $\tau_R(X)$ = {ϕ, {1, 3}, K}. Then $ng\mu$-closed sets are ϕ, {2}, {1, 2}, {2, 3}, K. Let L = {1, 2, 3} with L/ R'= {{1}, {2, 3}, {3, 2}} and Y= {2, 3}. Then nano topology $\tau'_R(Y)$ = {ϕ, {2, 3}, L}. Define f : (K, $\tau_R(X)$) → (L, $\tau'_R(Y)$) be the identity map. Then f is contra $ng\mu$-continuous but not contra nano continuous, since f^{-1}({2, 3})= {2, 3} is not nano closed in (K, $\tau_R(X)$).

4.15 Proposition

Every contra $ng\mu$-continuous is nano contra g-continuous but not conversely.

Proof Let f : (K, $\tau_R(X)$) → (L, $\tau'_R(Y)$) be a contra $ng\mu$-continuous map and let G be any nano open set in (L, $\tau'_R(Y)$). Then, f^{-1}(G) is $ng\mu$-closed in K. Since every $ng\mu$-closed set is ng-closed, f^{-1}(G) is ng-closed in K. Therefore f is nano contra g-continuous. □

4.16 Example

Let K = {1, 2, 3} with K/R = {{2}, {1, 3}} and X = {2}. Then nano topology $\tau_R(X)$ = {ϕ, {2}, K}. Then $ng\mu$-closed sets are ϕ, {1, 3}, K and ng-closed sets are ϕ, {1}, {3}, {1, 2}, {1, 3}, {2, 3}, K. Let L = {1, 2, 3} with L/ R'= {{1}, {2, 3}} and Y= {1, 3}. Then nano topology $\tau'_R(Y)$ = {ϕ, {1}, {2, 3}, L}. Define f : (K, $\tau_R(X)$) → (L, $\tau'_R(Y)$) be the identity map. Then f is nano contra g-continuous but not contra $ng\mu$-continuous, since $f^{-1}(\{2, 3\}) = \{2, 3\}$ is not $ng\mu$-closed in (K, $\tau_R(X)$).

4.17 Remark

$ng\mu$-continuity and contra $ng\mu$-continuity are independent.

4.18 Example

Let K, $\tau_R(X)$ and f be as in Example 4.14. Let L = {1, 2, 3} with L/ R'= {{3}, {1, 2}} and Y= {3}. Then nano topology $\tau'_R(Y) = \{\phi, \{3\}, L\}$. Then f is $ng\mu$-continuous but not contra $ng\mu$-continuous, since $f^{-1}(\{3\}) = \{3\}$ is not $ng\mu$-closed in (K, $\tau_R(X)$).

4.19 Example

Let K, $\tau_R(X)$, L, $\tau'_R(Y)$ and f be as in Example 4.14. Then f is contra $ng\mu$-continuous but not $ng\mu$-continuous, since $f^{-1}(\{1\}) = \{1\}$ is not $ng\mu$-closed in (K, $\tau_R(X)$).

4.20 Remark

The composition of two contra $ng\mu$-continuous maps need not be contra $ng\mu$-continuous.

4.21 Example

Let K, $\tau_R(X)$, L, $\tau'_R(Y)$ and f be as in Example 4.14. Then $ng\mu$-closed sets are ϕ, {1}, {1, 2}, {1, 3}, L}. Let M = {1, 2, 3} with M/ R*= {{1}, {2, 3}} and Z= {1}. Then nano topology $\tau^*_R(Z) = \{\phi, \{1\}, M\}$. Define g : (L, $\tau'_R(Y)$) → (M, $\tau^*_R(Z)$) be the identity map. Clearly f and g are contra $ng\mu$-continuous but their g ∘ f : (K, $\tau_R(X)$) → (M, $\tau^*_R(Z)$) is not contra $ng\mu$-continuous, because V = {1} is nano open in (M, $\tau^*_R(Z)$) but (g ∘ f $^{-1}(\{1\}) = f^{-1}(g^{-1}(\{1\}))$ = $f^{-1}(\{1\}) = \{1\}$, which is not $ng\mu$-closed in (K, $\tau_R(X)$).

4.22 Theorem

Let f : (K, $\tau_R(X)$) → (L, $\tau'_R(Y)$) be a map. Then the following conditions are equivalent

1. f is contra $ng\mu$-continuous.
2. The inverse image of each nano open set in P is $ng\mu$-closed in K.
3. The inverse image of each nano closed set in P is $ng\mu$-open in K.
4. For each point k in K and each nano closed set G in P with f(k) ∈ G, there is an $ng\mu$-open set U in K containing k such that f(U) ⊂ G.

Proof (1) ⇒ (2). Let G be nano open in L. Then L − G is nano closed in L. By definition of contra $ng\mu$-continuous, f^{-1}(L − G) is $ng\mu$-open in K. But f^{-1}(L − G) = K - f^{-1}(G). This implies f^{-1}(G) is $ng\mu$-closed in K.
(2) ⇒ (3) Let G be any nano closed set in L. Then L−G is nano open set in L. By the assumption of (2), f^{-1}(L − G) is $ng\mu$-closed in K. But f^{-1}(L − G) = K - f^{-1}(G). This implies f^{-1}(G) is $ng\mu$-open in K.
(3) ⇒ (4). Let k ∈ K and G be any nano-closed set in L with f(k) ∈ G. By (3), f^{-1}(G) is $ng\mu$-open in K. Set U= f^{-1}(G). Then there is an $ng\mu$-open set U in K containing k such that f(U) ⊂ G.
(4) ⇒ (1). Let k ∈ K and G be any nano-closed set in L with f(k) ∈ G. Then L − G is nano-open in L with f(k) ∈ G. By (4), there is an $ng\mu$-open set U in K containing k such that f(U) ⊂ G. This implies U= f^{-1}(G) . Therefore, K − U = K − f^{-1}(G) = f^{-1}(L−G) which is $ng\mu$-closed in K. □

4.23 Theorem

Let f : (K, $\tau_R(X)$) → (L, $\tau'_R(Y)$) and g : (L, $\tau'_R(Y)$) → (M, $\tau^*_R(Z)$). Then the following properties hold:

1. If f is contra $ng\mu$-continuous and g is nano continuous then g ∘ f is contra $ng\mu$-continuous.
2. If f is contra $ng\mu$-continuous and g is contra nano continuous then g ∘ f is $ng\mu$-continuous.
3. If f is $ng\mu$-continuous and g is contra nano continuous then g ∘ f is contra $ng\mu$-continuous.

Proof (1) Let G be nano closed set in M. Since g is nano continuous, g^{-1}(G) is nano closed in L. Since f is contra $ng\mu$-continuous, $(g \circ f)^{-1}$(G) = $f^{-1}(g^{-1}$(G)) is $ng\mu$-open in K. Therefore g ∘ f is contra $ng\mu$-continuous.
(2) Let G be any nano closed set in M. Since g is contra nano continuous, g^{-1}(G) is nano open in L. Since f is contra $ng\mu$-continuous, $(g \circ f)^{-1}$(G) = $f^{-1}(g^{-1}$(G)) is $ng\mu$-closed in K. Therefore g ∘ f is $ng\mu$-continuous.
(3) Let G be any nano closed set in M. Since g is contra nano continuous, g^{-1}(G) is nano open in L. Since f is $ng\mu$-continuous, $(g \circ f)^{-1}$(G) = $f^{-1}(g^{-1}$(G)) is $ng\mu$-open in K. Therefore g ∘ f is contra $ng\mu$-continuous. □

4.24 Theorem

Let f : (K, $\tau_R(X)$) → (L, $\tau'_R(Y)$) is $ng\mu$-irresolute map and g : (L, $\tau'_R(Y)$) → (M, $\tau^*_R(Z)$) is contra nano continuous map, then g ∘ f : (K, $\tau_R(X)$) → (M, $\tau^*_R(Z)$) is contra $ng\mu$-continuous map.
Proof Since g is contra nano continuous from (L, $\tau'_R(Y)$) → (M, $\tau^*_R(Z)$), for any nano open set in m as a subset of M, we get, g^{-1}(m) = G is a nano closed set in (L, $\tau'_R(Y)$). By Proposition 3.2, it implies that g^{-1}(m) = G is $ng\mu$-closed in (L, $\tau'_R(Y)$). As f is $ng\mu$-irresolute map. We get (g ∘ f)$^{-1}$(m)= f^{-1}(g^{-1}(m)) = f^{-1}(G) = S and S is a $ng\mu$-closed in (K, $\tau_R(X)$). Hence g ∘ f is a contra $ng\mu$-continuous map. □

4.25 Theorem

Let f : (K, $\tau_R(X)$) → (L, $\tau'_R(Y)$) is $ng\mu$-irresolute map and g : (L, $\tau'_R(Y)$) → (M, $\tau^*_R(Z)$) is contra $ng\mu$-continuous map, then g ∘ f : (K, $\tau_R(X)$) → (M, $\tau^*_R(Z)$) is contra $ng\mu$-continuous map.
Proof Since g is contra $ng\mu$-continuous from (L, $\tau'_R(Y)$) → (M, $\tau^*_R(Z)$), for any nano open set in m as a subset of M, we get, g^{-1}(m) = G is a $ng\mu$-closed set in (L, $\tau'_R(Y)$). As f is $ng\mu$-irresolute map. We get (g ∘ f)$^{-1}$(m)= f^{-1}(g^{-1}(m)) = f^{-1}(G) = S and S is a $ng\mu$-closed in (K, $\tau_R(X)$). Hence g ∘ f is a contra $ng\mu$-continuous map. □

4.26 Theorem

Let f : (K, $\tau_R(X)$) $\to$ (L, $\tau'_R(Y)$) be a map and g: (K, $\tau_R(X)$) $\to$ ((K, $\tau_R(X)$) $\times$ (L, $\tau'_R(Y)$)) the graph map of f, defined by g(k) = (k, f(k)) for every k $\in$ K. If g is contra $ng\mu$-continuous, then f is contra $ng\mu$-continuous.

Proof Let G be an nano open set in (L, $\tau'_R(Y)$). Then ((K, $\tau_R(X)$) $\times$ G) is an nano open set in ((K, $\tau_R(X)$) $\times$ (L, $\tau'_R(Y)$)). It follows from Theorem 4.22, that f^{-1}(G) = g^{-1}((K, $\tau_R(X)$) $\times$ G) is $ng\mu$-closed in (K, $\tau_R(X)$). Thus, f is contra $ng\mu$-continuous. □

Conclusions

In this paper, we offer a new class of sets called $ng\mu$-closed sets in nano topological spaces and we study some of its basic properties. We introduce and study $ng\mu$-continuous, $ng\mu$-irresolute and contra $ng\mu$-continuous. Moreover, we obtain their properties and characterizations. In future, we have extended this work in various nano topological fields with some applications.

Acknowledgement The authors would like to thank the editors and the anonymous reviewers for their valuable comments and suggestions which have helped immensely in improving the quality of the paper.

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