On soft pre-ω-open sets

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Abstract

In this paper, we used the notions of soft ω-open sets to introduce and study the concepts of soft pre-ω-open sets. Moreover, some of their properties are shown. Furthermore, the concepts of soft pre-T_1 and soft pre-T_2 spaces are defined.

Keywords: soft pre-ω-open sets, soft pre-T_1 spaces, soft pre-T_2 spaces.

2010 Mathematics Subject Classification: 54A40, 54A10, 54A05

1 Introduction

The notion of soft set was introduced by Molodtsov in 1999 [10], this concept is applied in several fields such that engineering, medical science, social science, etc. Besides, the concept of soft set has been grown by several researchers (see [9, 8]), El-Sheikh and El-Latif are ones of those authors who had studied this concept, they introduced the notion of semi-open soft set [5]. For a soft topological space (X, τ, A) and $(F, A) \in SS(X)_A$, then (F, A) is called pre-open soft set if $(F, A)\tilde{\subseteq} In(Cl(F, A))$. Besides, they proved some of their properties. On the other hand, Hdeib [7] introduced the notions of ω-closed sets as a weaker notion of closed set by: Let $(X, \mathfrak{F})$ be a topological space and $B \subset X$. A point $x \in X$ is called a condensation point of B if for each open set V with $x \in V$, the set $V \cap B$ is uncountable. B is said to be ω-closed if it contains all its condensation points. Many mathematicians have studied the concepts of ω-closed sets in different fields of the general topology (see [6, 3]). Recently, Al Ghour and Hamed introduced and studied the notions of soft ω-open sets and they proved some of their properties.

In this paper, motivated by the authors mentioned above, we extend the notion of soft ω-closed sets by using the concepts of soft pre-ω-open sets which are studied in this paper. Besides, we show some of their properties. Furthermore, we define the concepts of soft pre-T_1 and soft pre-T_2 spaces.

2 Soft pre-ω-open sets

In this section, we introduce and study the notion of soft pre-ω-open sets by using the concept of ω-set.

Definition 2.1. [4] Let $G \in SS(X, A)$. Then G is said to be a countable soft set if for all $a \in A$, the set $G(a)$ is countable. The collection of all countable soft sets from $SS(X, A)$ will be denoted by $CSS(X, A)$.

Definition 2.2. Let (X, τ, A) be a soft topological space and let $G \in SS(X\tau)$. Then, G is said to be soft pre-ω-open if for all $a_x\tilde{\in}G$, there exits a pre-open $F \in \tau$ and $H \in CSS(X, A)$ such that $a_x\tilde{\in}F - H\tilde{\subseteq}G$. The collection of all soft pre-ω-open sets in (X, τ, A) will be denoted by $\tau_{p\omega}$.

Theorem 2.3. *Let (X, τ, A) be a soft topological space and let $G \in SS(X, A)$. Then, G is soft pre-ω-open if and only if for every $a_x \tilde{\in} G$ there exits a pre-open $F \in \tau$ such that $a_x \tilde{F}$ and $F - G \in CSS(X, A)$.*

Proof. **Necessary**: Suppose that G is soft pre-ω-open and let $a_x \tilde{\in} G$, then there exits a pre-open $F \in \tau$ and $H \in CSS(X, A)$ such that $a_x \tilde{\in} F - H \tilde{\subseteq} G$. Hence, $a_x \tilde{\in} F \in \tau$. Besides, since $F - H \tilde{\subseteq} G$, we have that $F - G \tilde{\subseteq} H$ and so $F - G \in CSS(X, A)$.
Sufficiency: Suppose that for every $a_x \tilde{\in} G$, then there exits a pre-open $F \in \tau$ such that $a_x \tilde{\in} F$ and $F - G \in CSS(X, A)$. Let $a_x \tilde{\in} G$. Then, there exits a pre-open $F \in \tau$ such that $a_x \tilde{\in} F$ and $F - G \in CSS(X, A)$. Choose $H = F - (G \tilde{\cup} a_x)$. Therefore, $H \in CSS(X, A)$ and $a_x \tilde{\in} F - H \tilde{\subseteq} G$. Indeed, G is soft pre-ω-open. □

Remark 2.4. For a soft topological space (X, τ, A), we will denote the collection of $\{F - H : F \in \tau$, where F is a pre-open and $H \in CSS(X, A)\}$ by τ_{pc}.

Theorem 2.5. *Let (X, τ, A) be a soft topological space. Then, the following statements hold:*

1. *$\tau \subseteq \tau_{pc} \subseteq \tau_{p\omega}$.*
2. *$(X, \tau_{p\omega}, A)$ is a soft topological space.*
3. *τ_{pc} is a base for $\tau_{p\omega}$.*
4. *Countable soft sets are soft semi-closed in (X, τ_{pc}, A)*

Proof. 1. Since $0_A \in CSS(X, A)$, then $\tau = \{F - 0_A : F \in \tau$ and F is pre-open $\} \subseteq \tau_{sc}$. Now, $\tau_{pc} \subseteq \tau_{p\omega}$ is clear.

2. We will proof that $(X, \tau_{p\omega}, A)$ is a soft topological space, then:
   (a) Since $0_A, 1_A \in \tau$ and by part (1) of this Theorem, $0_A, 1_A \in \tau_{p\omega}$.
   (b) Let $F, G \in \tau_{p\omega}$ and let $a_x \tilde{\in} F \tilde{\cap} G$. Then, we have that $a_x \tilde{\in} F$ and $a_x \tilde{\in} G$. Then, by the Theorem 2.3, there exit pre-open sets $H, W \in \tau$ such that $a_x \tilde{\in} H \tilde{\cap} W \in \tau$ and $H - F, W - G \in CSS(X, A)$. So, we can see that $(H \tilde{\cap} W) - (F \tilde{\cap} G)$. In consequence, by the Theorem 2.3, $F \tilde{\cap} G \in \tau_{p\omega}$.
   (c) Let $\{G_\delta : \delta \in \Delta\} \subseteq \tau_{p\omega}$ and let $a_x \tilde{\in} \tilde{\bigcup}_{\delta \in \Delta} G_\delta$. Then, there is $\alpha \in \Delta$ such that $a_x \tilde{\in} G_\alpha$. Indeed, there exit pre-open $F \in \tau$ and $H \in CSS(X, A)$ such that $a_x \tilde{\in} F - H \tilde{\subseteq} G_\alpha \tilde{\subseteq} \tilde{\bigcup}_{\delta \in \Delta} G_\delta$. Therefore, $\tilde{\bigcup}_{\delta \in \Delta} G_\delta \in \tau_{p\omega}$.
3. This proof is clear.
4. The proof is followed by part (1) of this theorem by the fact $\tau_{pc} \subseteq \tau_{p\omega}$.

□

Definition 2.6. [2] Let (X, τ, A) be a soft topological space. Then, $\{0_A\} \cup \{1_A - H : H \in CSS(X, A)\}$ is called the cocountable soft topology and is denoted by $coc(X, A)$.

Proposition 2.7. *For any soft topological space (X, τ, A), $coc(X, A) \subseteq \tau_{pc}$.*

Proof. The proof is followed by the Remark 2.4 and Definition 2.6. □

Theorem 2.8. *Let (X, τ, A) be a soft topological space. Then, the following statements are equivalent:*

1. $coc(X, A) \subseteq \tau$.
2. $\tau = \tau_{pc}$.
3. $\tau = \tau_{p\omega}$.

Proof. (1) $\Rightarrow$ (2): Suppose that $coc(X, A) \tilde{\subseteq} \tau$. We have to show that $\tau_{pc} \subseteq \tau$. Now, let $G \in \tau$ a pre-open and $H \in CSS(X, A)$. Then, $G - H = G \tilde{\cap} (1_A - H)$. Since $coc(X, A) \subseteq \tau$ this implies that $1_A - H \in \tau$, where 1_A is pre-open and so $G - H \in \tau$. Hence, $\tau_{pc} \subseteq \tau$.
(2) $\Rightarrow$ (3): Suppose that $\tau = \tau_{pc}$. Then, τ_{pc} is a soft topology. Now, by the Theorem 2.5 part (3), we have that $\tau_{pc} = \tau_{p\omega}$ and then $\tau = \tau_{p\omega}$.
(3) $\Rightarrow$ (1): Suppose that $\tau = \tau_{p\omega}$. Then by the Proposition 2.7 and Theorem 2.5 part (1), we have that $coc(X, A) \subseteq \tau_{pc} \subseteq \tau_{p\omega} = \tau$. □

Proposition 2.9. *Let X be an initial universe and A be a set of parameters. Then, $(coc(X, A))_{p\omega} = coc(X, A)$.*

Proof. The proof follows. □

Theorem 2.10. *For any soft topological space (X, τ, A), $\tau_{p\omega} = (\tau_{p\omega})_{p\omega}$.*

Proof. By the Proposition 2.7 and Theorem 2.5 part (1), $coc(X, A) \subseteq \tau_{pc} \subseteq \tau_{p\omega}$. Then, by the Theorem 2.8, we have that $\tau_{p\omega} = (\tau_{p\omega})_{p\omega}$. □

Theorem 2.11. *Let (X, τ, A) and (X, σ, A) be two soft topological spaces. If $\tau \cup coc(X, A) \subseteq \sigma$, then $\tau_{pc} \subseteq \sigma$.*

Proof. Let $F - H \in \tau_{pc}$, where $F \in \tau$ is a pre-open and H is a countable soft set. Now, since $F \in \tau, 1_A - H \in coc(X, A)$ and $\tau \cup coc(X, A) \subseteq \sigma$. Indeed, $F, 1_A - H \in \sigma$ and then $F \tilde{\cap} (1_A - H) = F - H \in \sigma$. □

Lemma 2.12. *Let (X, τ, A) and (X, σ, A) be two soft topological spaces. If $\tau \cup coc(X, A) \subseteq \sigma$, then $\tau_{p\omega} \subseteq \sigma$.*

Proof. The proof is followed by the Theorems 2.11 and 2.5 part (3). □

Lemma 2.13. *[11] Let (X, τ, A) be a soft topological space and let $\mathfrak{B}$ be a soft base for τ. Then, for every $a \in A$, the family $\{F(a) : F \in \mathfrak{B}\}$ forms a base for the topology τ_a on X.*

Theorem 2.14. *Let (X, τ, A) be a soft topological space. Then, for all $a \in A$, $(\tau_a)_{p\omega} = (\tau_{p\omega})_a$.*

Proof. Let $a \in A$. We have to show that $(\tau_a)_{p\omega} \subseteq (\tau_{p\omega})_a$, it will be enough if we show that $U - C \in (\tau_{p\omega})_a$ for all $U \in \tau_a$ and a countable subset $C \subseteq X$. Now, let $U \in \tau_a$ and let C be a countable subset of X. Since $U \in \tau_a$, then there is a pre-open $F \in \tau$ such that $F(a) = U$. Let $H = a_C$, then $H \in CSS(X, A)$. So, we have that $F - H \in \tau_{p\omega}$, and then $(F - H)(a) = F(a) - H(a) = U - C \in (\tau_{p\omega})_a$.
Now, we have to show that $(\tau_{p\omega})_a \subseteq (\tau_a)_{p\omega}$. By the Theorem 2.5 and Lemma 2.13, we only have to prove that $\{(F - H)(a) : F \in \tau$, where F is pre-open and $H \in CSS(X, A)\} \subseteq (\tau_a)_{p\omega}$. Let $F \in \tau$, where F is pre-open and $H \in CSS(X, A)$, then $(F - H)(a) = F(a) - H(a)$ with $F(a) \in \tau_a$ and $H(a)$ is a countable set of X and this implies that $(F - H)(a) \in (\tau_a)_{p\omega}$. □

Proposition 2.15. *Let (X, τ, A) be a soft topological space. If $G \in \tau_{p\omega}$, then for all $a \in A$, $G(a) \in (\tau_a)_{p\omega}$.*

Proof. Let $G \in \tau_{p\omega}$ and let $a \in A$. Then, $G(a) \in (\tau_{p\omega})_a$ and by the Theorem 2.14, $G(a) \in (\tau_a)_{p\omega}$. □

Lemma 2.16. *[1] Let X be an initial universe and let A be a set of parameters. Let $\{\mathfrak{F}_a : a \in A\}$ be an indexed family of topologies on X. If $\mathfrak{B}_a$ is a base for $\mathfrak{F}_a$ for all $a \in A$. Then, $\{a_{\mathfrak{Y}} : a \in A$ and $\mathfrak{Y} \in \mathfrak{B}_a\}$ is a soft base of $\bigoplus_{a\in A} \mathfrak{F}_a$.*

Theorem 2.17. *Let X be an initial universe and let A be a set of parameters. Let $\{\mathfrak{F}_a : a \in A\}$ be an indexed family of topologies on X. Then, $(\bigoplus_{a\in A} \mathfrak{F}_a)_{p\omega} = \bigoplus_{a\in A}(\mathfrak{F}_a)_{p\omega}$.*

Proof. We begin proving that $(\bigoplus_{a\in A} \mathfrak{F}_a)_{p\omega} \subseteq \bigoplus_{a\in A}(\mathfrak{F}_a)_{p\omega}$. By the Theorem 2.5 part (3), it will be enough showing $(\bigoplus_{a\in A} \mathfrak{F}_a)_{pc} \subseteq \bigoplus_{a\in A}(\mathfrak{F}_a)_{p\omega}$. Now, let $F \in \bigoplus_{a\in A} \mathfrak{F}_a$, where F is pre-open and H is a countable soft set. Then, for every $a \in A$, $F(a)\imath\mathfrak{F}_a$ and $H(a)$ is a countable subset of X and then $(F - H)(a) = F(a) - H(a) \in (\mathfrak{F}_a)_{p\omega}$. Indeed, $F - H \in \bigoplus_{a\in A}(\mathfrak{F}_{\mathfrak{a}})_{p\omega}$ for every $a \in A$, $\{U - C : U \in \mathfrak{F}_a$ and C is a countable subset of $X\}$ is a base for $(\mathfrak{F}_a)_{p\omega}$, by the Lemma 2.16, $\{a_{U-C} : a \in A, U \in \mathfrak{F}_a$ and C is countable subset of $X\}$ is a soft base for $\bigoplus_{a\in A}(\mathfrak{F}_a)_{p\omega}$. Hence, if we will prove that $\bigoplus_{a\in A}(\mathfrak{F}_a)_{p\omega} \subseteq (\bigoplus_{a\in A} \mathfrak{F}_a)_{p\omega}$, it is sufficient to show that $\{a_{U-C} : a \in A, U \in \mathfrak{F}_a$ and C is a countable subset of $X\} \subseteq (\bigoplus_{a\in A} \mathfrak{F}_a)_{p\omega}$. Now, we can see that $\{a_{U-C} : a \in A, U \in \mathfrak{F}_a$ and C is a countable subset of $X\} = \{a_{U-C} - a_C : a \in A, U \in \mathfrak{F}_a$ and C is a countable subset of $X\}$, and so, this ends the proof. □

Lemma 2.18. *[1] If $(X, \mathfrak{F})$ is a topological space and A is any ser of parameters, then $(\tau(\mathfrak{F}))_a = \mathfrak{F}$ for all $a \in A$.*

Proposition 2.19. *If $(X, \mathfrak{F})$ is a topological space and A is any set of parameters, then $(\tau(\mathfrak{F}))_{p\omega} = \tau(\mathfrak{F}_{p\omega})$ for all $a \in A$.*

Proof. For each $a \in A$, the set $\mathfrak{F}_a = \mathfrak{F}$. Then, $\tau(\mathfrak{F}) = \bigoplus_{a\in A} \mathfrak{F}_a$ and by the Theorem 2.17, we have that:

$$(\tau(\mathfrak{F}))_{p\omega} = (\bigoplus_{a\in A} \mathfrak{F}_a)_{p\omega}$$

$$= \bigoplus_{a\in A}(\mathfrak{F}_a)_{p\omega}$$

$$= \tau(\mathfrak{F}_{p\omega}).$$

□

Definition 2.20. Let (X, τ, A) be a soft topological space. Then, (X, τ, A) is said to be soft pre-p-space if the countable intersection of soft pre-open sets is soft pre-open.

Definition 2.21. [4] Let $F \in SS(X, A)$. F is called a soft point over X relative to A if there exit $e \in A$ and $x \in X$ such that

$$F(a) = \begin{cases} \{x\} & \text{if } a = e \\ \emptyset & \text{if } a \neq e \end{cases}$$

We denote F by e_x. The family of all soft points over X relative to A is denoted by $SP(X, A)$.

Definition 2.22. Let (X, τ, A) be a soft topological space. Then, (X, τ, A) is said to be soft pre-T_1 if for any two soft points $a_x, a_y \in SP(X, A)$ with $x \neq y$, there exit pre-open sets $G, F \in \tau$ such that $a_x \tilde{\in} G - F$ and $a_y \tilde{\in} F - G$.

Lemma 2.23. *A soft topological space (X, τ, A) is soft pre-T_1 if for every soft point $a_x \in SP(X, A)$ is soft pre-closed.*

Proof. The proof is followed by the Definition 2.22 □

Theorem 2.24. *If (X, τ, A) is soft pre-T_1 and soft pre-p-space, then $\tau = \tau_{p\omega}$.*

Proof. By the Theorem 2.5 part (1), $\tau \subseteq \tau_{p\omega}$. Now, to show that $tau_{p\omega} \subseteq \tau$, by the Theorem 2.5 part (3), it is enough to show that $\tau_{pc} \subseteq \tau$. Let $F \in \tau$, where F is pre-open and let $HCSS(X, A)$. Since (X, τ, A) is soft pre-T_1, then by the Lemma 2.23, a_x soft closed for all $a_x \tilde{\in} H$, and so $F - a_x \in \tau$ for all $a_x \tilde{\in} H$, where $F - a_x$ is pre-open. Since (X, τ, A) is soft pre-T_1, then $\tilde{\bigcap}_{a_x \in H} (F - a_x) \in \tau$ is pre-open. Therefore, $F - H = \tilde{\bigcap}_{a_x \in H} (F - a_x)$. □

Lemma 2.25. *For any soft topological space (X, τ, A). $(X, \tau_{p\omega}, A)$ is soft pre-T_1.*

Proof. The proof is followed by the Theorem 2.5 part (4) and Lemma 2.23. □

Definition 2.26. Let (X, τ, A) be a soft topological space. Then, (X, τ, A) is said to be soft pre-T_2 if for two soft points $a_x, a_y \in SP(X, A)$ with $x \neq y$, there exit pre-open sets $G, F \in \tau$ such that $a_x \tilde{\in} G$, $a_y \tilde{\in} F$ and $G \tilde{\cap} F = 0_A$.

Theorem 2.27. *If (X, τ, A) is a soft topological space and pre-T_2. Then, $(X, \tau_{s\omega}, A)$ is soft pre-T_2.*

Proof. Let $a_x, a_y \in SP(X, A)$ with $x \neq y$. Since (X, τ, A) is soft pre-T_2, then there exit pre-open sets $G, F \in \tau$ such that $a_x \tilde{\in} G$, $a_y \tilde{\in} F$ and $G \tilde{\cap} F = 0_A$. Now, by the Theorem 2.5 part (1), $\tau \subseteq \tau_{p\omega}$ and hence $G, F \in \tau_{p\omega}$ an this ends the proof. Therefore, $(X, \tau_{p\omega}, A)$ is soft pre-T_2. □

3 Conclusion

In this paper we extended a new notions of soft open sets by using the concept of soft ω-closed set [2], in that paper the authors studied whole the notions related it. For that reason, we took those notions and showed other results by using soft pre-open sets and soft ω-closed sets, these sets were namely soft pre ω-open which were studied in the the above section. For further studies, we recommend to find out other notions in which involve the sets studied in this paper, besides it will be possible to study some variants of continuity through soft pre-ω-open sets.

Acknowledgement

We want to thank referees for their useful suggestions which helped us to improve our manuscript.

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