

Possibility to Realize an Accelerated Turing Machine

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Abstract

On the basis of a theorem, in which an evanescent photon is a superluminal particle, the author considers the possibility of realizing a high-performance computer system compared with conventional silicon processors. To realize such a quantum computer utilizing evanescent photons, we must replace electronic components with optical ones, and thus an equivalent optical transistor is required. This critical component for quantum computing can be created using meta-material circuits with a non-linear refractive index. Based on this optical computer system, utilizing metamaterial technology, it can be shown that superluminal computation, which is a new concept for an accelerated Turing machine, can be realized in the physical world.

Keywords: Accelerated Turing Machine, Zeno Machine, Superluminal Particles, Hypercomputation

1. Introduction

In mathematics and computer science, an accelerated Turing machine is a hypothetical computational model related to Turing machines, which can perform a countable infinite number of computational steps within a finite time. It is also called a Zeno machine, a concept proposed independently by B. Russdel, R. Blake and H. Weyl, which performs its first computational step in one unit of time and each subsequent step in half the time of the step before, allowing an infinite number of steps of computation to be completed within a finite interval of time [1,2]. However, such a machine cannot be physically realized, from the standpoint of the Heisenberg uncertainty principle, because the energy to perform the computation will be exponentially increasing as the computational speed is accelerating. Thus, the Zeno machine is considered a mere mathematical concept with no possibility to realize it in the physical world.

Contrary to this conclusion, the author can show the possibility to realize a real accelerated Turing machine, by utilizing superluminal particles instead of subluminal particles, which are related to evanescent photons, and also a metamaterial circuit.

2 Computational Time Required to Perform Infinite Steps of Computation

Hypercomputation, the term of which was introduced in 1999 by Jack Copeland and Diane Proudfoot [3], that can complete infinite computation steps, refers to models of computation that go beyond, or are incomparable to Turing computability. This includes various hypothetical methods for the computation of non-Turing-computable functions. The Church–Turing thesis states that any function that is algorithmically computable can be computed by a Turing machine. An accelerated Turing machine is included in the category

of hypercomputer. It can compute functions that a Turing machine cannot and which are not computable in the Church-Turing sense.

Feynmann defined the required energy per step for a computation as shown in Figure 1, given by [4]

$$energy\ per\ step = k_B T \frac{f - b}{(f + b)/2} \tag{1}$$

where k_B is Boltzmann's constant, T is a temperature, f is a forward rate of computation and b is backward rate.

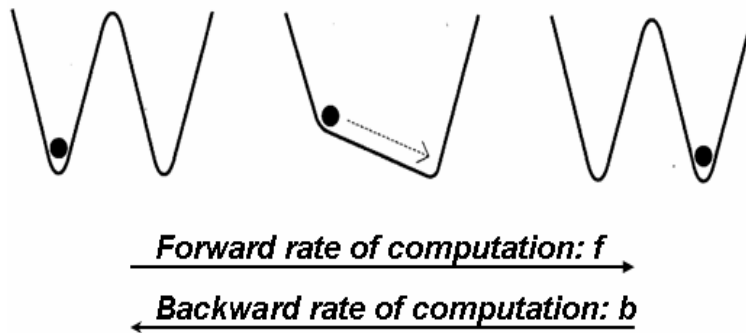


Figure 1. Computational steps necessary to perform reversible computation.

Supposing that there is no energy supply and parameters f and b are fixed during the computation, we can consider the infinite computational steps given by

$$E_1 = kE_0, E_2 = kE_1, \dots, E_n = kE_{n-1}, \dots \tag{2}$$

where we let the initial energy of computation be $E_0 = k_B T$, $k = 2(f - b)/(f + b)$, and E_n is the energy for the n-th step computation.

From the above, we have $E_n = k^n E_0$, and the energy loss for each computational step becomes

$$\begin{aligned} \Delta E_1 &= E_0 - E_1 = (1 - k)E_0 \\ \Delta E_2 &= E_1 - E_2 = (1 - k)kE_0 \\ &\vdots \end{aligned} \tag{3}$$

$$\Delta E_n = E_{n-1} - E_n = (1-k)k^{n-1}E_0$$

E. Recami claimed in his paper [5] that tunneling photons traveling in evanescent mode can move with superluminal group speed inside the barrier. Chu and S. Wong at AT&T Bell Labs measured superluminal velocities for light traveling through the absorbing material [6]. Furthermore Steinberg, Kwait and Chiao devised an experiment measuring the tunneling time for visible light through an optical filter, consisting of a multilayer coating about 10^{-6} m thick, and confirmed superluminal speed [7]. The results obtained by Steinberg and co-workers have shown that the photons seemed to have traveled at 1.7 times the speed of light. Recent optical experiments at Princeton NEC have verified that superluminal pulse propagation can occur in transparent media [8]. These results indicate that the process of tunneling in quantum physics is indeed superluminal, as claimed by E. Recami.

From the relativistic equation of energy and momentum of the moving particle, given by

$$E = \frac{m_0 c^2}{\sqrt{1 - v^2 / c^2}}, \quad (4)$$

and

$$p = \frac{m_0 v}{\sqrt{1 - v^2 / c^2}}, \quad (5)$$

the relation between the energy and momentum can be shown as $p/v = E/c^2$.

From which, we have [9]

$$\frac{v\Delta p - p\Delta v}{v^2} = \frac{\Delta E}{c^2}. \quad (6)$$

Supposing the approximation $\Delta v/v^2 \approx 0$ holds, Equation (6) can be simplified as

$$\Delta p \approx \frac{v}{c^2} \Delta E. \quad (7)$$

This relation is also valid for a superluminal particle, which has an imaginary mass im_* .

According to the paper by M. Park and Y. Park [10], the uncertainty relation for a superluminal particle can be given by

$$\Delta p \cdot \Delta t \approx \frac{\hbar}{v - v'}, \tag{8}$$

where v and v' are the velocities of the superluminal particle after and before the measurement. By substituting Equation (7) into (8), we obtain the uncertainty relation for superluminal particles given by

$$\Delta E \cdot \Delta t \approx \frac{\hbar}{\beta(\beta - 1)}, \tag{9}$$

when we let $v' = c$ and $\beta = v/c$.

From Equation (4), we have the following equation for superluminal particles;

$$E_n = \frac{m_* c^2}{\sqrt{v^2/c^2 - 1}}, \tag{10}$$

where E_n is the energy for the n-th step computation and m_* is a proper mass of the superluminal particle.

From which, the speed of superluminal particles from Equation (2) becomes

$$v/c = \sqrt{1 + \frac{m_*^2 c^4}{k^{2n} E_0^2}}, \tag{11}$$

where n is the step number of the computation.

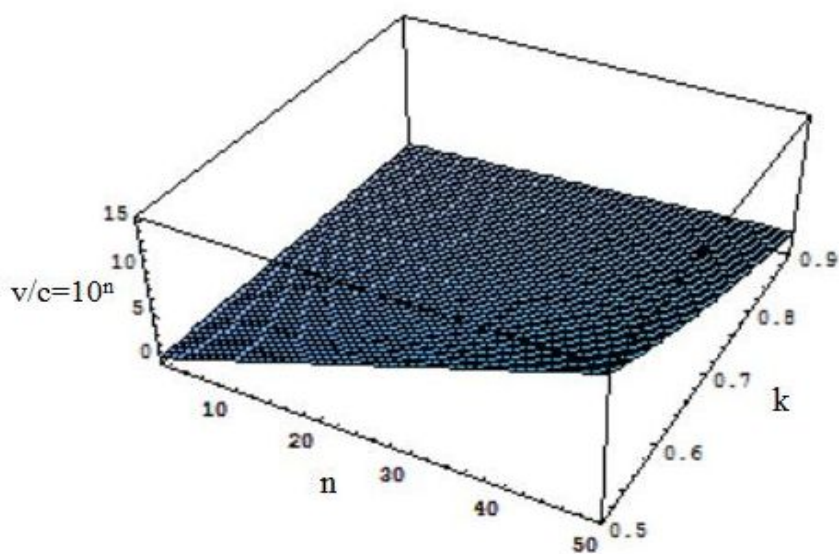


Figure.2 Computational speed per steps ($m_* c^2 / E_0 = 1$)

From this equation, the speed of superluminal particles can be obtained as shown in Figure.2 with the parameters of k and n .

From this figure, it is seen that the computational speed is accelerated per steps.

Hence the total time required for the quantum system utilizing superluminal particles becomes [9]

$$T = \sum_{n=1}^{\infty} \Delta t_i = \frac{\pi \hbar}{2E_0} \sum_{n=1}^{\infty} \frac{1}{\beta_n (\beta_n - 1)(1-k)k^{n-1}}, \tag{12}$$

from Equation (3) and the uncertainty principle for superluminal particles given by Equation (9), where β_n can be given by

$$\beta_n = \sqrt{1 + \frac{m_*^2 c^4}{E_n^2}} = \sqrt{1 + \frac{m_*^2 c^4}{k^{2n} E_0^2}}, \tag{13}$$

which is derived from Equation (10).

From Equations (12) and (13), it is seen that the computation time can be given as shown in Figure 3.

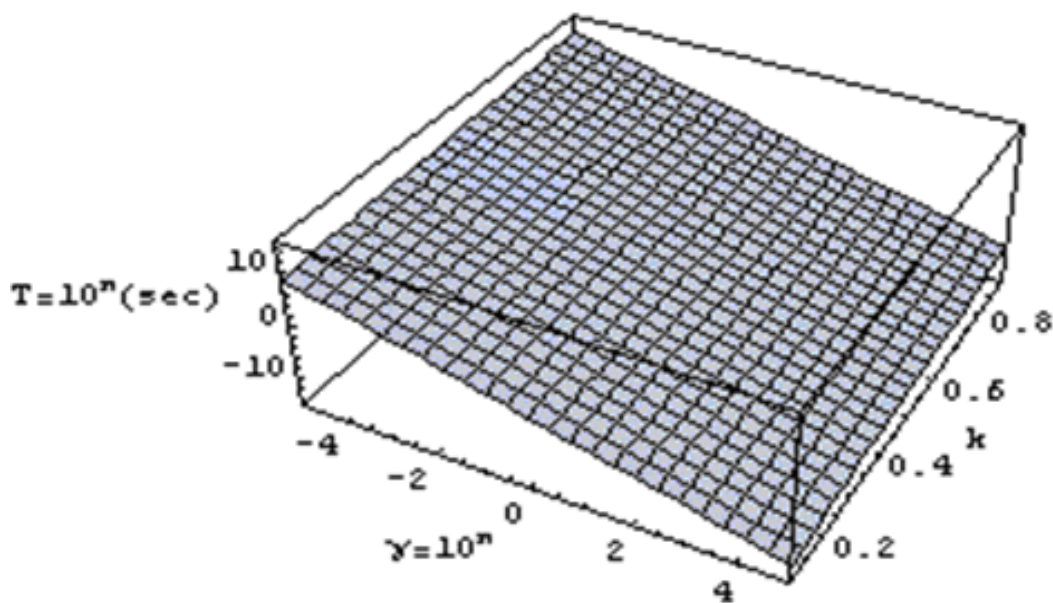


Figure 3. Computational required time for superluminal particles.

By numerical calculation, it can be shown that the infinite sum of Equation (12) converges to a certain value satisfying $0 < k < 1$, as shown in Figure 3.

In this figure, the horizontal line is for the parameter $\gamma = m_*c^2 / E_0$ and the vertical line is for the time to complete infinite calculation steps.

These calculation results indicate that an accelerated Turing machine can be realized, utilizing superluminal particles instead of subluminal particles, in accordance with Feynman's computational model.

Thus, contrary to the conclusion that an accelerated Turing machine cannot be physically realized from the standpoint of the Heisenberg uncertainty principle, it can be seen that superluminal particles permit the construction of a real, accelerated Turing machine.

3. Decoherence Time of Computation by Superluminal Particles

According to Zurek, the density matrix $\rho(x, x')$ of the particle in the position representation evolves by the master equation shown as [11]

$$\dot{\rho} = -\frac{i}{\hbar}[H, \rho] - \gamma(x - x') \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'} \right) \rho - \frac{2m\gamma k_B T}{\hbar^2} (x - x')^2 \rho, \quad (14)$$

where H is the particle's Hamiltonian, γ is the relaxation rate, k_B is the Boltzmann constant, and T is the temperature of the field.

Zurek has shown that the off-diagonal peaks of the density matrix, $\rho(x, x') = \varphi(x)\varphi^*(x')$ will decay at the rate

$$\frac{d}{dt}(\rho^\pm) \approx 2\gamma m k_B T / \hbar^2 (\Delta x)^2 \rho^\pm = \rho^\pm / \tau_D. \quad (15)$$

From which, quantum coherence will disappear on a time scale shown by [12]

$$\tau_D \cong \tau_R \left(\frac{\hbar}{\Delta x \sqrt{2m k_B T}} \right)^2, \quad (16)$$

where Δx is a wave packets separation and τ_R is the relaxation time.

The decoherence time for a quantum computation utilizing superluminal particles, has been shown by the author using the formula [13];

$$\tau'_D / \tau_D \approx [\beta(\beta - 1)]^2, \quad (17)$$

where τ'_D is the decoherence time for the superluminal particle.

In the following formula β is the ratio of the velocity of a superluminal particle relative to the speed of light;

$$\beta \approx 1 + \frac{c}{2\omega d} + \sqrt{\frac{c}{\omega d} + \frac{c^2}{4\omega^2 d^2}}, \quad (18)$$

where ω is an angular frequency of the particle and d is the size of the tunneling barrier.

When we utilize an evanescent wave with frequencies in the infra-red spectrum, which wavelength is given by

$\lambda = 100\mu m$, we have $\tau'_D / \tau_D = 1.3 \times 10^{12}$ for the case when we let $d = 15nm$.

Thus, the decoherence time utilizing evanescent photons in a quantum processor is given by

$$\tau'_D \approx \tau_R \left(\frac{\hbar}{\Delta x \sqrt{2mk_B T}} \right)^2 [\beta(\beta - 1)]^2 \approx 9.82 \times 10^2 \tau_R \text{ (sec)}, \quad (19)$$

where we suppose the effective mass of an evanescent photon to be $m = 1.78 \times 10^{-35} \text{ Kg}$, $T = 300^\circ K$ and $\Delta x = 1.0cm$.

For the case when the error rate is small enough, it may be possible to use quantum error correction, which corrects errors due to decoherence, thereby allowing the total calculation can be completed less than the decoherence time.

If we suppose that the required error rate in each gate is 10^{-4} , this implies that each gate must be able to perform its task in one 10,000th of the decoherence time of the system.

We suppose that the relaxation time was an order of macroscopic time satisfying $\tau_R \gg 1 \text{ sec}$, each gate can perform its task within 0.1 second and it is enough for each gate in a quantum computer system to complete its computation task.

4. Computer Architecture for Superluminal Computation

From the assumption that the evanescent photon is a superluminal particle, the author has shown that a tubule structure constructed by metamaterial can achieve quantum bit (qubit) computations on large data sets, which would account for the high performance of the computations as compared with the conventional processors, with very low energy compared with the conventional silicon processors [14].

It therefore seems highly plausible that macroscopic quantum ordered dynamical systems of evanescent photons in the processor could play an essential role in realizing long-range coherence in a computing system.

Ziolkowski pointed out in his paper that superluminal pulse propagation, which permits consequent superluminal exchange without a violation in causality, is possible in electromagnetic metamaterials [15].

These metamaterials achieve desired effects by incorporating structural elements of sub-wavelength sizes, i.e. features that are actually smaller than the wavelength of the waves.

Negative refractive index materials appear to permit the creation of super lenses that can have a spatial resolution below that of the wavelength. If the inner medium of a cylinder of metamaterial tube possesses the characteristics of a negative refractive index, the generation of evanescent photons is enhanced, and they propagate without loss inside the tubes according to the properties of a metamaterial [16].

Metamaterial consists of a periodic structure with the period being small compared to the optical wavelength, which affects electromagnetic waves by having structural features smaller than the wavelength of the respective electromagnetic wave.

They are particularly strong for frequencies near the resonance frequency.

The most interesting optical effects may occur somewhat above or below the resonance.

This can create a negative permeability and hence it can be seen from the following relation between k , ϵ_r , and μ_r ;

$$k^2 = \epsilon_r \mu_r \omega^2 / c^2, \quad (20)$$

where ϵ_r and μ_r respectively indicate the relative electric and magnetic permittivity, that the wave number inside the tube-like structure becomes imaginary which can produce evanescent wave enhancement.

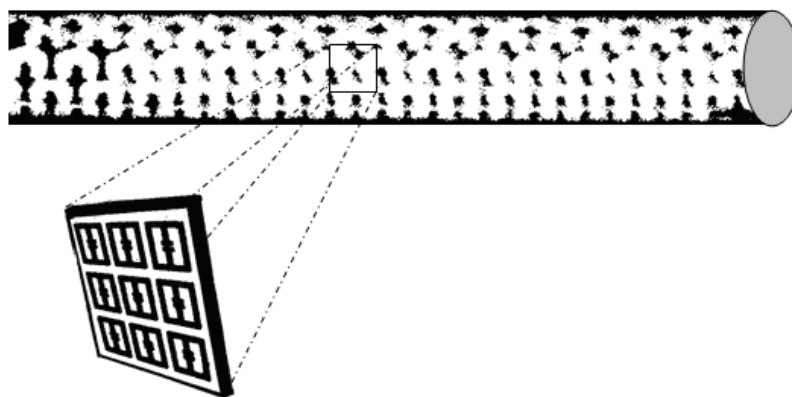


Figure. 4 Structure of the wave guide of superluminal particles

When a photon impinges such a structure composed of nano-resonators as shown in Figure 4, it can excite electromagnetic oscillations.

As shown in this figure, the optical logic gate consists of a periodic structure with the period being small compared to the optical wavelength and it affects electromagnetic waves by having structural features smaller than the wavelength of the respective electromagnetic wave.

B.-I. Popa and S. A. Cummer confirmed the evanescent wave enhancement inside passive metamaterial structure [17] by their experiment and we can conclude that the waveguide shown in Figure.4 can propagate evanescent photons inside it by its metamaterial structure. To replace electronic components with optical ones, an equivalent optical transistor is required. This is achieved using materials with a non-linear refractive index. In particular, materials exist where the intensity of incoming light affects the intensity of the light transmitted through the material in a similar manner to the voltage response of an electronic transistor [18]. Such an "optical transistor" [19, 20] can be used to create optical logic gates, which in turn are assembled into the higher-level components of the computer's CPU. These will be non-linear crystals used to manipulate light beams into controlling others.

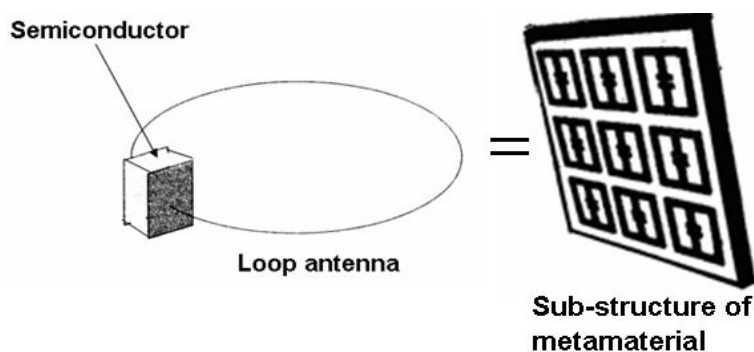


Figure 5. Optical logic gate composed of metamaterial.

Photonic logic is the use of photons (light) in logic gates such as NOT, AND, OR, NAND, and etc. Switching is obtained using nonlinear optical effects when two or more signals are combined.

Resonators are especially useful in photonic logic, since they allow a build-up of energy from constructive interference, thus enhancing optical nonlinear effects by using semiconductors inside the sub-structure of metamaterial.

Figure 7 shows the schematic diagram of superluminal computation, which is composed of an evanescent photon generator, the quantum processor and the holographic memory. The basic idea of evanescent wave holography is to use an evanescent wave as the reference wave and as the read-out wave. The readout wave is diffracted by the hologram structure inside the cylindrical structure of a storage material and the object field can be constructed [21]. This is the same mechanism of holographic memory within the brain. The ability to store and recover holographic information provides a mechanism for optical memory.

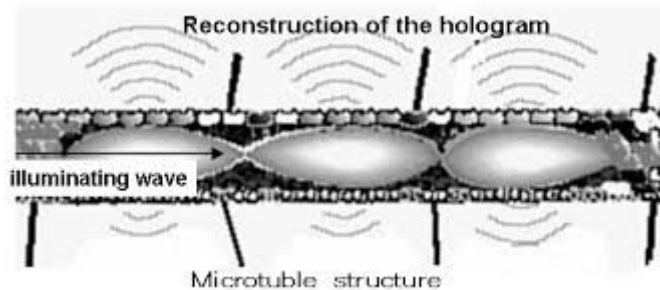


Figure.6 Holographic memory within the brain

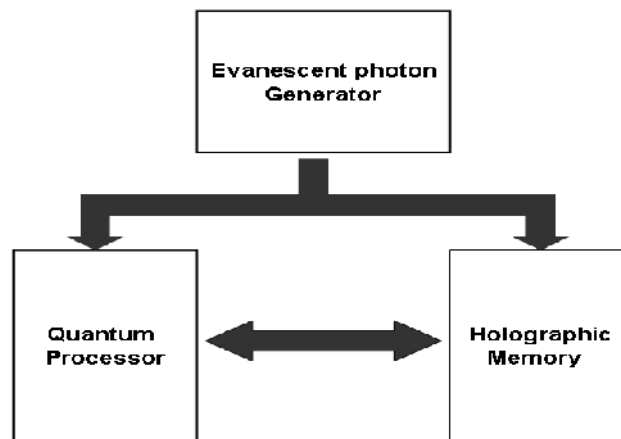


Figure 7. Schematic diagram of superluminal computing.

The quantum processor and the holographic memory can be constructed from substructures of metamaterial inside the tube-like structure.

An evanescent wave is a near-field wave with an intensity that exhibits exponential decay without absorption as a function of the distance from the boundary at which the wave is formed. Evanescent photons are formed

when waves traveling in a medium undergo total internal reflection at its boundary because they strike it at an angle greater than the so-called critical angle. For example, this evanescent photon field can be generated as shown by Figure 8.

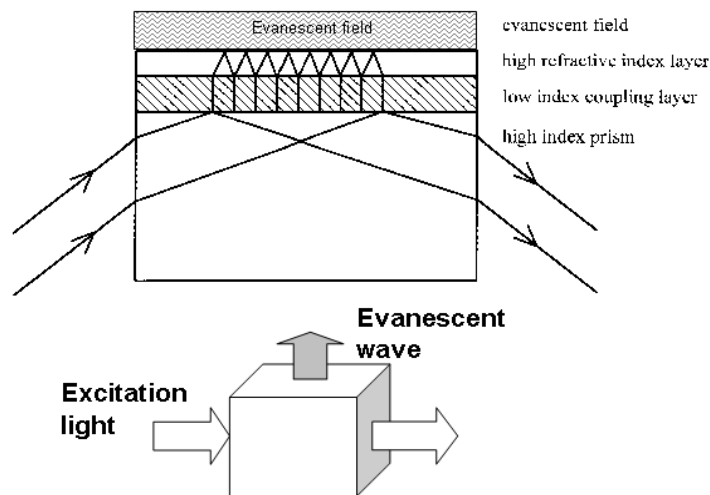


Figure 8. Evanescent photon generator.

Evanescent waves are formed at the boundary between two media with different wave motion properties, and are most intense within one third of a wavelength from the surface of formation. They are supplied to tube-like structures as shown in Figure.8 by the coupling which is usually accomplished by placing two or more electromagnetic elements such as optical waveguides close together so that the evanescent field generated by one element does not decay much before it reaches the other element. With these waveguides, the evanescent field gives rise to propagating-wave modes, thereby connecting the wave from one waveguide to the next. Mathematically, the process is the same as that of quantum tunneling.

From the above theoretical analysis, it is seen that we can realize an accelerated Turing machine (which can be called a hypercomputer), which requires much less energy for the operation than the conventional silicon processors by utilizing evanescent photons.

Silicon has given us the computers which we have today by allowing billions of transistors to be packed onto a single chip. Now, researchers have demonstrated a silicon chip that manipulates individual photons to create a quantum photonic processor.

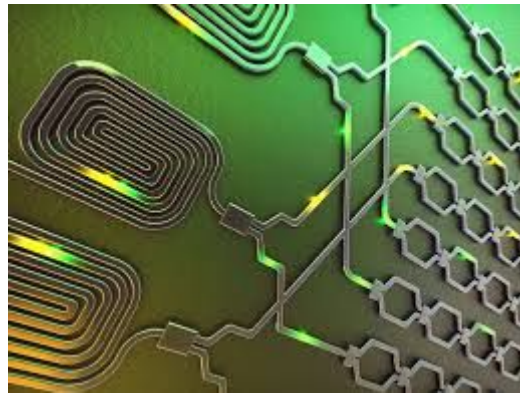


Figure.9 Quantum photonic processor [22]

This chip consists of many interferometers, which split the photons into different spatial modes. Each mode passes through a specific waveguide, so having a photon in one waveguide represents a 1, while in another it represents a 0. The photons are encoded using thermo-optical phase shifters, which are controlled by electrical voltages. If we can figure out a way to generate many more identical, entangled photons on the chip and fitting enough phase shifters, beam splitters, and other optical components onto the chip to handle all those photons, we have the practical way to implement the ultimate large-scale photonic quantum processor in a small box size.

By utilizing this photonic computer technology and we use evanescent photon and metamaterial tube instead of photons and optical fiber, we can construct a hypercomputer which is more powerful than a quantum computer according the theory presented above in this paper.

5. Discussions and Conclusion

It is known that an accelerated Turing machine could allow us to compute functions which are not Turing-computable, such as the halting problem [23], described as “given a description of an arbitrary computer program, decide whether the program finishes running or continues to run forever”.

A Turing machine cannot decide if an arbitrary program halts or runs forever. Some proposed that hypercomputers can simulate the program for an infinite number of steps and tell the user whether the program is halted or not. This is equivalent to the problem of deciding, given a program and an input, whether the program will eventually halt when run with that input, or will run forever.

Halting problem for Turing machines can be easily solved by an accelerated Turing machine using the following pseudocode algorithm shown as [24];

```
begin program
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write 0 on the first position of the output tape;
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begin loop

simulate 1 successive step of the given Turing machine on the given input;

if the Turing machine has halted,

then write 1 on the first position of the output tape

and break out of loop;

end loop

end program.

As an accelerated Turing machine is more powerful than ordinary Turing machines, they can perform computation beyond the Turing limit, which is called hypercomputation, such as to decide any arithmetic statement that is infinite time decidable. From this result, we can construct an oracle machine, which is an abstract machine used to study decision problems, by using a superluminal particle.

A Turing machine with a black box, called an oracle, would be able to decide certain decision problems in a single operation. For example, if the problem is a decision problem for a set of natural numbers, the Turing machine supplies the oracle with a natural number, and the oracle responds with "yes" or "no" stating whether that number is an element of this set.

Given a device that tells you in advance whether a given computer program would halt, or go on running forever, you would be able to prove or disprove any integer theorem for enormous large numbers.

Hence, we can see that there does exist the possibility to realize a hypercomputational system with low energy cost, like a human brain, if we utilize superluminal particles and metamaterial structure. As an accelerated Turing machine, which can be called hypercomputer, is more powerful than ordinary Turing machines, and it can perform computation beyond the Turing limit such as to decide any arithmetic statement that is decidable in an infinite amount of time.

If we can utilize superluminal particles, i.e. evanescent photons instead of ordinary particles and metamaterial circuits, it will one day be possible to build a hyper computer system which has a great potentiality compared with conventional computer systems.

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