Pythagorean Neutrosophic Pre-Open Sets

Carlos Granados
Corporación Universitaria Latinoamericana
carlosgranadosortiz@outlook.es

Abstract

The purpose of this paper is to introduce and study the notion of Pythagorean neutrosophic pre-open sets by using the notion of Pythagorean neutrosophic open set. Besides, we define the concepts of Pythagorean neutosophic pre-open function, Pythagorean neutrosophic pre-continuous function and Pythagorean neutrosophic pre-homeomorphism. Moreover, some of their properties are proved.

Keywords: Pythagorean neutrosophic pre-open sets, Pythagorean neutosophic pre-open function, Pythagorean neutrosophic pre-continuous function, Pythagorean neutrosophic pre-homeomorphism.

2010 Mathematics Subject Classification: Primary: 54A40, 03E72, Secondary 03E99.

1 Introduction and preliminaries

The notion of fuzzy set was introduced by Zadeh [12] and then this notion has been studied by many mathematicians in different fields of the general topology (see [6, 4]). In 1968, Chang [5] introduced the notion of fuzzy topological spaces, as well as, some basics concepts in general topology. Besides, Atanassov [2, 3] in 1983 defined the concept of intuitionistic fuzzy set. Furthermore, the notion of neutrosophic set was introduced by Smarandache [9] and so Wang et.al. studied some of its properties on interval neutrosophic set. Moreover, the notion of neutrosophic topological space was defined by Salama and Albowi [8]. By using the notions mentioned above, Yager [11] in 2013 introduced the concept of Pythagorean membership grades, later Yager, Zahand and Xu [10] proved some properties on Pythagorean fuzzy set. On the other hand, in 2017 Arockiarani [1] introduced and studied the notion of neutrosophic pre-open set, Besides, Shena and Nirmala [7] introduced the notion of Pythagorean neutrosophic open sets and showed some properties on Pythagorean neutrosophic α-open set.

In this paper, we used the notion of Pythagorean neutrosophic open set to introduce and study the concept of Pythagorean neutrosophic pre-open set, besides we show some of its properties. We also define the concepts of Pythagorean neutosophic pre-open function, Pythagorean neutrosophic pre-continuous function and Pythagorean neutrosophic pre-homeomorphism. Moreover, some of their properties are proved.

Throughout this paper, $(X, \mathfrak{T})$, $(Y, \mathfrak{Y})$ and $(Z, \mathfrak{Z})$ are topological spaces on which no separation axioms are assumed unless otherwise mentioned. Furthermore, we sometimes write X, Y or Z instead of $(X, \mathfrak{T})$, $(Y, \mathfrak{Y})$ or $(Z, \mathfrak{Z})$, respectively. Now, we show some Definitions which are useful for the developing of this paper.

Definition 1.1. [12] A fuzzy set $A = \{\langle x, \mu_A(x)\rangle : x \in X\}$ is a universe of discourse X, which is characterized by a membership function μ_A as $\mu_A : X \to [0, 1]$.

Definition 1.2. [2, 3] Let X be a non-empty set. Then, A is said to be an intuitionistic fuzzy set of X if there is a $A = \{\langle x, \mu_A, \gamma_A\rangle : x \in X\}$ where the function $\mu_A : X \to [0, 1]$ and $\gamma_A : X \to [0, 1]$ denote the degree of

CC BY

membership $\mu_A(x)$ and degree of non-membership γ_A of every element $x \in X$ to the set A and satisfies the condition $0 \leq \mu_A(x) + \gamma_A(x) \leq 2$.

Definition 1.3. [9] Let X be a non-empty set. Then, A is said to be a neutrosophic set of X if there is a $A = \{\langle x, \mu_A, \sigma_A, \gamma_A \rangle : x \in X\}$ where the function $\mu_A : X \to [0,1]$, $\sigma_A : X \to [0,1]$ and $\gamma_A : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$), degree of indeterminacy (namely $\sigma_A(x)$) and degree of non-membership (namely $\gamma_A(x)$) of each element $x \in X$ to the set A and satisfies the condition $0 \leq \mu_A(x) + \sigma_A(x) + \gamma_A(x) \leq 3$.

Definition 1.4. [11] Let X be a universal set. Then, a Pythagorean fuzzy set A, which is a set of ordered pairs on X and it is defined by $A = \{\langle x, \mu_A(x), \gamma_A(x) \rangle : x \in X\}$ where the function $\mu_A : X \to [0,1]$ and $\gamma_A : X \to [0,1]$ define the degree of membership and the degree of non-membership respectively , of the element $x \in X$ to A, which is subsets in X and for every $x \in X : 0 \leq (\mu_A(x))^2 + (\gamma_A(x))^2 \leq 1$. Assuming that $0 \leq (\mu_A(x))^2 + (\gamma_A(x))^2 \leq 1$, there is a degree of indeterminacy of $x \in X$ to A defined by $\pi_A(x) = \sqrt{(\mu_A(x))^2 + (\gamma_A(x))^2}$ and $\pi_A(x) \in [0,1]$.

Definition 1.5. [7] Let X be a non-empty set. Then, A is said to be a Pythagorean neutrosophic set (or simply, PN) of X if there is a $A = \{\langle x, \mu_A, \sigma_A, \gamma_A \rangle : x \in X\}$ where the function $\mu_A : X \to [0,1]$, $\sigma_A : X \to [0,1]$ and $\gamma_A : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$), degree of indeterminacy (namely $\sigma_A(x)$) and degree of non-membership (namely $\gamma_A(x)$) of each element $x \in X$ to the set A and satisfies the condition $0 \leq \mu_A(x)^2 + \sigma_A(x)^2 + \gamma_A(x)^2 \leq 1$.

Definition 1.6. [7] A Pythagorean neutrosophic topology (or simply, PNT) on a non-empty set X is a family of $\mathfrak{T}$ of Pythagorean neutrosophic sets in X satisfying the following conditions:

1. $0, 1 \in \mathfrak{T}$.
2. $\mathfrak{G}_1 \cap \mathfrak{G}_2 \in \mathfrak{T}$, for any $\mathfrak{G}_1, \mathfrak{G}_2 \in \mathfrak{T}$.
3. $\bigcup \mathfrak{G}_i \in \mathfrak{T}$, for any arbitrary family $\{\mathfrak{G}_i : \mathfrak{G}_i \in \mathfrak{T}, i \in I\}$.

In this case, the pair $(X, \mathfrak{T})$ is said to be a Pythagorean neutrosophic topological spaces, besides any Pythagorean neutrosophic set in $\mathfrak{T}$ is known as Pythagorean neutrosophic neutrosophic open set in X.

Definition 1.7. For a Pythagorean neutrosophic set A in a Pythagorean neutrosophic topological space $(X, \mathfrak{T})$ is said to be Pythagorean neutrosophic α-open set [7] if $A \subseteq PNInt((PNCl(PNInt(A)))$.

Theorem 1.8. *[7] Every Pythagorean neutrosophic open set is Pythagorean neutrosophic α-open set.*

Definition 1.9. [7] Let $f : (X, \mathfrak{T}) \to (Y, \mathfrak{Y})$ be a function where $(X, \mathfrak{T})$ and $(Y, \mathfrak{Y})$ are Pythagorean neutrosophic topological spaces. Then, f is said to be Pythagorean neutrosophic if $f^{-1}(V)$ is a Pythagorean neutrosophic in X for every Pythagorean neutrosophic open set V in Y.

Definition 1.10. [7] Let $f : (X, \mathfrak{T}) \to (Y, \mathfrak{Y})$ be a function where $(X, \mathfrak{T})$ and $(Y, \mathfrak{Y})$ are Pythagorean neutrosophic topological spaces. Then, f is said to be Pythagorean neutrosophic α-continuous if $f^{-1}(V)$ is a Pythagorean neutrosophic α-open in X for every Pythagorean neutrosophic open set V in Y.

Definition 1.11. [7] Let $f : (X, \mathfrak{T}) \to (Y, \mathfrak{Y})$ be a function where $(X, \mathfrak{T})$ and $(Y, \mathfrak{Y})$ are Pythagorean neutrosophic topological spaces. Then, f is said to be Pythagorean neutrosophic α-open if $f(A)$ is Pythagorean neutrosophic α-open set in Y for every Pythagorean neutrosophic open set A in X.

2 Pythagorean neutrosophic pre-open sets

In this section we introduce and study the notion of Pythagorean neutrosophic pre-open set and we show some of its properties.

Definition 2.1. Let X be a non-empty set. If a, b, c are real standard or non standard subsets of $]0^-, 1^+[$, then the Pythagorean neutrosophic set $x_{a,b,c}$ is said to be Pythagorean neutrosophic point (or simply, PNP) in X and it is given by:

$$x_{a,b,c}(x_p) = \begin{cases} (a, b, c) & \text{if } x = x_p \\ (0, 0, 1) & \text{if } x \neq x_p \end{cases}$$

For each $x_p \in X$ is said to be the support of $x_{a,b,c}$, where a denotes the degree of membership value, b denotes the degree of indeterminacy and c is the degree of non-membership value of $x_{a,b,c}$.

Definition 2.2. For a Pythagorean neutrosophic set A in a Pythagorean neutrosophic topological space $(X, \mathfrak{T})$ is said to be Pythagorean neutrosophic pre-open set (or simply, $PNpOS$) if $A \subseteq PNInt((PNCl(A))$. The complement of a Pythagorean neutrosophic pre-open set is called Pythagorean neutrosophic pre-closed set.

Remark 2.3. The collection of all Pythagorean neutrosophic pre-open sets and Pythagorean neutrosophic pre-closed sets are denoted by $PNpOS(X, \mathfrak{T})$ and $PNpCS(X, \mathfrak{T})$, respectively.

Proposition 2.4. *Let $(X, \mathfrak{T})$ be a Pythagorean neutrosophic topological space and $A \subseteq X$. Then, If A is a Pythagorean neutrosophic α-open set, then A is Pythagorean neutrosophic pre-open set.*

Proof. Let A be a Pythagorean neutrosophic α-open, then by the Definition 1.7, $A \subseteq PNInt(PNCl(PNInt(A)))$, since $Int(A) \subseteq A$, this implies that $A \subset PNInt(PNCl(A))$. Therefore, A is Pythagorean neutrosophic pre-open. □

Definition 2.5. A Pythagorean neutrosophic set V in a Pythagorean neutrosophic topological space $(X, \mathfrak{T})$ is said to be Pythagorean neutrosophic pre-closed (or simply, $PNpCS$)) if $V \supseteq PNInt(PNCl(V))$.

Definition 2.6. Let $(X, \mathfrak{T})$ be a Pythagorean neutrosophic topological space and V be a Pythagorean neutrosophic set on X. Then we define the Pythagorean neutrosophic pre-interior and Pythagorean neutrosophic pre-closure of V as:

1. Pythagorean neutrosophic pre-interior of V (or simply, $PNPINT(V)$) as the union of all Pythagorean neutrosophic pre-open sets of X contained in V. It means that $PNPINT(V) = \bigcup\{A : A$ is a $PNpOS$ in X and $A \subseteq V\}$.
2. Pythagorean neutrosophic pre-closure of V (or simply, $PNPCL(V)$) as the intersection of all Pythagorean neutrosophic pre-closed set of X containing V. It means that $PNPCL(V) = \bigcap\{B : B$ is a $PNpCS$ in X and $V \subseteq B\}$.

Remark 2.7. By the Definition 2.6, we can see that $PNPCL(V)$ is the smallest Pythagorean neutrosophic pre-closed set of X which contains V. Besides, $PNPINT(V)$ is the largest Pythagorean neutrosophic pre-open set of X which is contained in V.

Proposition 2.8. *Let V be a Pythagorean neutrosophic set in a Pythagorean neutrosophic topological space $(X, \mathfrak{T})$. Then, the following statements hold:*

1. *If V is Pythagorean neutrosophic pre-open set, then $Cl(V)$ is is a Pythagorean neutrosophic pre-closed set.*
2. *If V is Pythagorean neutrosophic pre-closed set, then $Cl(V)$ is is a Pythagorean neutrosophic pre-open set.*

Proof. The proof is followed by the Definitions 2.2, 2.5 and 2.6. □

Theorem 2.9. *Let V be a Pythagorean neutrosophic set in a Pythagorean neutrosophic topological space $(X, \mathfrak{T})$. Then, the following statements hold:*

1. $Cl(PNPINT(V)) = PNPCL(Cl(V))$.
2. $Cl(PNPCL(V)) = PNPINT(Cl(V))$.

Proof. We begin proving (1): Let V be a Pythagorean neutrosophic set. Now, by the Definition 2.6 part (1), $PNPINT(V) = \bigcup\{A : A$ is a $PNpOS$ in X and $A \subseteq V\}$, this implies that $Cl(PNPINT(V)) = Cl(\bigcup\{A : A$ is a $PNpOS$ in X and $A \subseteq V\})= \bigcap\{Cl(A) : Cl(A)$ is a $PNpCS$ in X and $Cl(V) \subseteq Cl(A)\}$. Now, we will replace $Cl(A)$ by B, then we have that $Cl(PNPINT(V) = \bigcap\{B : B$ is a $PNpCS$ in X and $Cl(V) \subseteq B\}$, and so $Cl(PNPINT(V)) = PNPCL(Cl(V))$.

The proof of (2) is similar to (1). □

Theorem 2.10. *For a Pythagorean neutrosophic topological space $(X, \mathfrak{T})$ and $A, B \subseteq X$. The following statements hold:*

1. *Every Pythagorean neutrosophic set is Pythagorean neutrosophic pre-open set.*
2. $PNPINT(PNPINT(A)) = PNPINT(A)$.
3. $PNPCL(PNPCL((A)) = PNPCL(A)$.
4. *Let A, B be two Pythagorean neutrosophic pre-open sets, then $PNpOS(A) \cup PNpOS(B) = PNpOS(A \cup B)$.*
5. *Let A, B be two Pythagorean neutrosophic pre-closed sets, then $PNpCS(A) \cap PNpCS(B) = PNpCS(A \cap B)$.*
6. *For any two sets A, B, $PNPINT(A) \cap PNPINT(B) = PNpint(A \cap B)$.*
7. *For any two sets A, B, $PNPCL(A) \cup PNPCL(B) = PNPCL(A \cup B)$.*
8. *If A is $PNpOS(X, \tau)$, then $A = PNPINT(A)$.*
9. *If $A \subseteq B$, then $PNPINT(A) \subseteq PNPINT(B)$.*
10. *For any two sets A, B, $PNPINT(A) \cup PNPINT(B) \subseteq PNPINT(A \cup B)$.*
11. *If A is $PNpCS(X, \tau)$, then $A = PNPCL(A)$.*
12. *If $A \subseteq B$, then $PNPCL(A) \subseteq PNPCL(B)$.*
13. *For any two sets A, B, $PNPCL(A \cap B) \subseteq PNPCL(A) \cap PNPCL(B)$.*

Proof. The proofs of (1), (2), (3), (4), (5), (9), (11) and (12) are followed by the Definitions 2.2 and 2.5. The proofs of (6), (7) and (8) are followed by the Definition 2.6 and the proofs of (10) and (13) are followed by the Definition 2.6 and parts (9) and (12) of this Theorem, □

The following example shows that the intersection of two Pythagorean neutrosophic pre-open sets need not be a Pythagorean neutrosophic pre-open set.

Example 2.11. Let $X = \{q, w\}$, $\mathfrak{A} = \langle(0.1, 0.3, 0.5), (0.3, 0.5, 0.7)\rangle$, $\mathfrak{B} = \langle(0.1, 0.1, 0.4), (0.7, 0.5, 0.3)\rangle$, $\mathfrak{C} = \langle(0.4, 0.6, 0.9), (0.6, 03, 0.3)\rangle$ and $\mathfrak{D} = \langle(0.3, 0.5, 0.3), (0.9, 0.5, 0.9)\rangle$. Then, $\mathfrak{T}$ is a Pythagorean neutrosophic topological space. Now, choose $\mathfrak{A}_1 = \langle(0.3, 0.5, 0.3), (1.0, 0.1, 0.1)\rangle$ and $\mathfrak{A}_2 = \langle(1.0, 1.0, 0.4), (0.9, 0.4, 0.6)\rangle$. We can see that $\mathfrak{A}_1 \cap \mathfrak{A}_2$ is not a Pythagorean neutrosophic pre-open set of $(X, \mathfrak{T})$.

The following example shows that the union of two Pythagorean neutrosophic pre-closed sets need not be a Pythagorean neutrosophic pre-closed set.

Example 2.12. By the example 2.11, we can imply that $\mathfrak{A}_1^c \cup \mathfrak{A}_2^c$ is not a Pythagorean neutrosophic pre-closed set of $(X, \mathfrak{T})$.

Proposition 2.13. *Let A be a Pythagorean neutrosophic set in Pythagorean neutrosophic topological space $(X, \mathfrak{T})$. If B is a Pythagorean neutrosophic pre-open set and $B \subseteq A \subseteq PNInt(PNCl(A))$, then A is a Pythagorean neutrosophic pre-open set.*

Proof. Let B be a Pythagorean neutrosophic pre-open set, then by the Definition 2.2, $B \subseteq PNInt(PNCl(B))$, and so $B \subseteq A \subseteq PNInt(PNCl(B)) \subseteq PNInt(PNCl(A))$. In consequence, A is a Pythagorean neutrosophic pre-open set □

Theorem 2.14. *Arbitrary union of Pythagorean neutrosophic pre-open sets is a Pythagorean neutrosophic pre-open set.*

Proof. Let $A_1, A_2, ...A_n$ be a collection of Pythagorean neutrosophic pre-open sets, then by the Definition 2.2, $A_1 \subseteq PNInt(PNCl(A_1))$, $A_2 \subseteq PNInt(PNCl(A_2))$, ..., $A_n \subseteq PNInt(PNCl(A_n))$. Now, $A_1 \cup A_2 \cup ... \cup A_n \subseteq PNInt(PNCl(A_1)) \cup PNInt(PNCl(A_2)) \cup ... \cup PNInt(PNCl(A_n))$, by the Theorem 2.10 parts (7) and (10), $A_1 \cup A_2 \cup ... \cup A_n \subseteq PNInt(PNCl(A_1 \cup A_2 \cup ... \cup A_n))$. This proofs that $A_1 \cup A_2 \cup ... \cup A_n$ is a Pythagorean neutrosophic pre-open set. □

Remark 2.15. By the Example 2.11, the arbitrary intersection of Pythagorean neotrosophic pre-open sets need not be a Pythagorean neotrosophic pre-open set.

Proposition 2.16. *Arbitrary intersection of Pythagorean neutrosophic pre-closed sets is a Pythagorean neutrosophic pre-closed set.*

Proof. The proof is followed by the Theorem 2.14 and parts (6) and (13) of the Theorem 2.10. □

Remark 2.17. By the Example 2.12, the arbitrary union of Pythagorean neotrosophic pre-closed sets need not be a Pythagorean neotrosophic pre-closed set.

Theorem 2.18. *A Pythagorean neutrosophic set A in a Pythagorean neutrosophic topological space $(X, \mathfrak{T})$ is Pythagorean neutrosophic pre-open if and only for every Pythagorean neutrosophic point $x_{a,b,c} \in A$ there exits a Pythagorean neutrosophic pre-open $B_{x_{a,b,c}}$ such that $x_{a,b,c} \in B_{x_{a,b,c}} \subseteq A$.*

Proof. **Necessary**: Let A be a Pythagorean neutrosophic pre-open set. Then, we have that $B_{x_{a,b,c}} = A$ for each $x_{a,b,c}$.

Sufficiency: Suppose that for every Pythagorean neutrosophic point $x_{a,b,c} \in A$, tehre exits a neutrosophic pre-open set $B_{x_{a,b,c}}$ such that $x_{a,b,c} \in B_{x_{a,b,c}} \subseteq A$. Thus, $A = \bigcup\{x_{a,b,c} : x_{a,b,c} \in A\} \subseteq \{B_{x_{a,b,c}} : x_{a,b,c} \in A\} \subseteq A$ and then, $A = \bigcup\{B_{x_{a,b,c}} : x_{a,b,c} \in A\}$. Therefore, by the Theorem 2.14, it is a Pythagorean neutrosophic pre-open set □

Definition 2.19. Let $f : (X, \mathfrak{T}) \to (Y, \mathfrak{Y})$ be a function where $(X, \mathfrak{T})$ and $(Y, \mathfrak{Y})$ are Pythagorean neutrosophic topological spaces. Then, f is said to be Pythagorean neutrosophic pre-open if $f(A)$ is Pythagorean neutrosophic pre-open set in Y for every Pythagorean neutrosophic open set A in X.

Proposition 2.20. *Let $f : (X, \mathfrak{T}) \to (Y, \mathfrak{Y})$ be a function where $(X, \mathfrak{T})$ and $(Y, \mathfrak{Y})$ are Pythagorean neutrosophic topological spaces. If f is Pythagorean neutrosophic α-open, then f is Pythagorean neutrosophic pre-open.*

Proof. Let f be a Pythagorean neutrosophic α-open and A be a Pythagorean neutrosophic open set in X. Then, by hypothesis $f(A)$ is a Pythagorean neutrosophic α-open set in Y, by the Proposition 2.4, $f(A)$ is a Pythagorean neutrosophic pre-open set in X. Therefore, f is a Pythagorean neutrosophic pre-open function. □

3 Pythagorean neutrosophic pre-continuous functions

In this section we used the notion of Pythagorean neutrosophic pre-open set to introduce and study the concepts of Pythagorean neutrosophic pre-continuous function and Pythagorean neutrosophic pre-homeomorphism, as well as, some of their properties are shown.

Definition 3.1. Let $f : (X, \mathfrak{T}) \to (Y, \mathfrak{Y})$ be a function where $(X, \mathfrak{T})$ and $(Y, \mathfrak{Y})$ are Pythagorean neutrosophic topological spaces. Then, f is said to be Pythagorean neutrosophic pre-continuous if $f^{-1}(V)$ is a Pythagorean neutrosophic pre-open set in X for every Pythagorean neutrosophic open set V in Y.

Proposition 3.2. *Every Pythagorean neutrosophic continuous function is Pythagorean neutrosophic pre-continuous function.*

Proof. The proof is followed by the Definition 1.8 and Proposition 2.4. □

Proposition 3.3. *For a function $f : (X, \mathfrak{T}) \to (Y, \mathfrak{Y})$, the following statements are equivalent:*

1. *f is Pythagorean neutrosophic pre-continuous.*
2. *$f^{-1}(V)$ is Pythagorean neutrosophic pre-closed set in X for every Pythagorean neutrosophic closed set V in Y.*
3. *$PNCl(PNIt(f^{-1}(V))) \subseteq f^{-1}(PNCl(V))$ for every Pythagorean neutrosophic set V in Y.*

Proof. (1) ⇒ (2): It is followed by the Definition 3.1.

(2) ⇒ (3): Let A be a Pythagorean neutrosophic set in Y. Then, $PNCl(A)$ is a Pythagorean neutrosophic closed set. Now, by hypothesis, $f^{-1}(PNCl(A))$ is a Pythagorean neutrosophic pre-closed set in X and so $PNCl(PNInt(f^{-1}(A))) \subseteq PNCl(PNInt(f^{-1}(PNCl(A))))) \subseteq f^{-1}(PNCl(A))$.

(3) $\Rightarrow$ (1): Let A be a Pythagorean neutrosophic open set of Y. Then, $Cl(A)$ is a Pythagorean neutrosophic closed set of Y. Thus, $PNCl(PNInt(f^{-1}(Cl(A)))) \subseteq f^{-1}(PNCl(Cl(A))) = f^{-1}(A)$. Indeed, $Cl(PNInt(PNCl(f^{-1}(A)))) = f^{-1}(A)$ and hence $Cl(PNInt(PNCl(f^{-1}(A)))) = PNCl(PNInt(f^{-1}(Cl(A)))) \subseteq f^{-1}(Cl(A)) = Cl(f^{-1}(A))$, this implies that $f^{-1}(A) \subseteq PNInt(PNCl(f^{-1}(A)))$. Therefore, $f^{-1}(A)$ is a Pythagorean neutrosophic pre-open set of X and by the Definition 3.1, f is a Pythagorean neutrosophic pre-continuous function. □

Definition 3.4. Let $x_{a,b,c}$ be a Pythagorean neutrosophic point of a Pythagorean neutrosophic topological space $(X, \mathfrak{T})$. A Pythagorean neutrosophic set D of X is said to be Pythagorean neutrosophic neighbourhood of $x_{a,b,c}$ if there exits a Pythagorean neutrosophic open set V in X such that $x_{a,b,c} \in V \subseteq D$

Proposition 3.5. *Let $f : (X, \mathfrak{T}) \to (Y, \mathfrak{Y})$ be a function where $(X, \mathfrak{T})$ and $(Y, \mathfrak{Y})$ are Pythagorean neutrosophic topological spaces. Then, the following statements are equivalent:*

1. *f is a Pythagorean neutrosophic pre-continuous function.*
2. *For each Pythagorean neutrosophic point $x_{a,b,c}$ and every Pythagorean neutrosophic A of $f(x_{a,b,c})$, there exits a Pythagorean neutrosophic pre-open set B of X such that $x_{a,b,c} \in B \subseteq f^{-1}(A)$.*
3. *For each Pythagorean neutrosophic point $x_{a,b,c} \in X$ and every Pythagorean neutrosophic neighbourhood A of $f(x_{a,b,c})$, there exits a Pythagorean neutrosophic pre-open set B of X such that $x_{a,b,c} \in B$ and $f(B) \subseteq A$.*

Proof. (1) $\Rightarrow$ (2): Let $x_{a,b,c}$ be a Pythagorean neutrosophic point of X and let A be a Pythagorean neutrosophic neighbourhood of $f(x_{a,b,c})$. Then, there exits a Pythagorean neutrosophic open set B of Y such that $f(x_{a,b,c}) \in B \subseteq A$. Now, since f is a Pythagorean neutrosophic pre-continuous function, we have that $f^{-1}(B)$ is a Pythagorean neutrosophic pre-open set of X and $x_{a,b,c} \in f^{-1}(f(x_{a,b,c})) \subseteq f^{-1}(B) \subseteq f^{-1}(A)$ and this ends the proof.

(2) $\Rightarrow$ (3): Let $x_{a,b,c}$ be a Pythagorean neutrosophic point of X and let A be a Pythagorean neutrosophic neighbourhood of f($x_{a,b,c}$). By hypothesis, there exits a Pythagorean neutrosophic pre-open set B of X such that $x_{a,b,c} \in B \subseteq f^{-1}(A)$ and then $x_{a,b,c} \in B$ of X such that $f(B) \subseteq f(f^{-1}(A)) \subseteq A$ and this ends the proof.

(3) $\Rightarrow$ (1): Let B be a Pythagorean neutrosophic open set of Y and let $x_{a,b,c} \in f^{-1}(B)$ and so $f(x_{a,b,c}) \in B$ and then B is a Pythagorean neutrosophic neighbourhood of $f(x_{a,b,c})$. Now, since B is a Pythagorean neutrosophic open set and by hypothesis, there exits a Pythagorean neutrosophic pre-open set A of X such that $x_{a,b,c} \in A$ and $f(A) \subseteq B$. Indeed, $x_{a,b,c} \in A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(B)$ and this implies that $f^{-1}(B)$ is a Pythagorean neutrosophic pre-open set of X. Therefore, f is a Pythagorean neutrosophic pre-open continuous function. □

Proposition 3.6. *Let $f : (X, \mathfrak{T}) \to (Y, \mathfrak{Y})$ be a function where $(X, \mathfrak{T})$ and $(Y, \mathfrak{Y})$ are Pythagorean neutrosophic topological spaces. If f is a Pythagorean neutrosophic α-continuous function, then f is a Pythagorean neutrosophic pre-open function.*

Proof. The proof is followed by the Definitions 1.11, 3.1 and Proposition 2.4. □

Definition 3.7. Let $f : (X, \mathfrak{T}) \to (Y, \mathfrak{Y})$ be a bijection function where $(X, \mathfrak{T})$ and $(Y, \mathfrak{Y})$ are Pythagorean neutrosophic topological spaces. Then, f is said to be Pythagorean nuetrosophic pre-homeomorphism if f and f^{-1} are Pythagorean neutrosophic pre-continuous functions.

Example 3.8. Let $X = \{q, w\}$ and $Y = \{e, r\}$. Then, $\mathfrak{T} = \{0_N, \mathfrak{U}_1, \mathfrak{U}_2, 1_N\}$ and $\mathfrak{Y} = \{0_N, \mathfrak{V}, 1_N\}$ are Pythagorean neutrosophic topological spaces on X and Y respectively, where $\mathfrak{U}_1 = \langle x, (0.2, 0.4, 0.7), (0.4, 0.4, 0.4)\rangle$, $\mathfrak{U}_2 = \langle x, (0.3, 0.5, 0.6), (0.5, 0.4, 0.6)\rangle$ and $\mathfrak{V} = \langle y, (0.3, 0.5, 0.6), (0.5, 0.2, 0.7)\rangle$. Then, we define the function $f : (X, \mathfrak{T}) \to (Y, \mathfrak{Y})$ as $f(q) = e$ and $f(w) = w$. We can see that f and f^{-1} are Pythagorean neutrosophic pre-continuous and then f is Pythagorean neutrosophic pre-homeomorphism.

Definition 3.9. Let $f : (X, \mathfrak{T}) \to (Y, \mathfrak{Y})$ be a bijection function where $(X, \mathfrak{T})$ and $(Y, \mathfrak{Y})$ are Pythagorean neutrosophic topological spaces. Then, f is said to be Pythagorean nuetrosophic homeomorphism if f and f^{-1} are Pythagorean neutrosophic continuous functions.

Theorem 3.10. *Each Pythagorean neutrosophic homeomorphism is Pythagorean neustrosophic pre-homeomorphism.*

Proof. Let $f : (X, \mathfrak{T}) \to (Y, \mathfrak{Y})$ be a bijection and Pythagorean neutrosophic homeomorphism function in which f and f^{-1} are Pythagorean neutrosophic continuous functions. Since that every Pythagorean neutrosophic continuous function is Pythagorean neutrosophic pre-continuous, this implies that f and f^{-1} are Pythagorean neutrosophic pre-continuous functions. Therefore, f is a Pythagorean neutrosophic pre-homeomorphism. □

The following example shows that the converse of the above Theorem need not be true.

Example 3.11. Let $X = \{q, w\}$ and $Y = \{e, r\}$. Then, $\mathfrak{T} = \{0_N, \mathfrak{U}_1, \mathfrak{U}_2, 1_N\}$ and $\mathfrak{Y} = \{0_N, \mathfrak{V}, 1_N\}$ are Pythagorean neutrosophic topological spaces on X and Y respectively, where $\mathfrak{U}_1 = \langle x, (0.3, 0.5, 0.8), (0.4, 0.4, 0.4)\rangle$, $\mathfrak{U}_2 = \langle x, (0.1, 0.3, 0.8), (0.1, 0.5, 0.8)\rangle$ and $\mathfrak{V} = \langle y, (0.4, 0.5, 0.6), (0.1, 0.3, 0.6)\rangle$. Then, we define the function $f : (X, \mathfrak{T}) \to (Y, \mathfrak{Y})$ as $f(q) = e$ and $f(w) = w$. We can see that f is a Pythagorean neutrosophic pre-homeomorphism, but it is not a Pythagorean neutrosophic homeomorphism.

Theorem 3.12. *Let $f : (X, \mathfrak{T}) \to (Y, \mathfrak{Y})$ be a bijection function where $(X, \mathfrak{T})$ and $(Y, \mathfrak{Y})$ are Pythagorean neutrosophic topological spaces. Then, the following statements hold:*

1. *f is Pythagorean neutrosophic pre-closed.*
2. *f is Pythagorean neutrosophic pre-open.*
3. *f is Pythagorean neutrosophic pre-homeomorphism.*

Proof. $(1) \Rightarrow (2)$: Let f be a bijection Pythagorean neutrosophic pre-closed function. Then, f^{-1} is Pythagorean neutrosophic pre-continuous function. Now, since every Pythagorean neutrosophic open set of $(X, \mathfrak{T})$ is a Pythagorean neutrosophic pre-open set of $(X, \mathfrak{T})$, this implies that f is a Pythagorean neutrosophic pre-open function.

$(2) \Rightarrow (3)$: Let f be a bijective Pythagorean neutrosophic pre-open function. Then, f^{-1} is a Pythagorean neutrosophic pre-continuous function. Indeed, f and f^{-1} are Pythagorean neutrosophic pre-continuous functions. Therefore, f is a Pythagorean neutrosophic pre-homeomorphism.

$(3) \Rightarrow (1)$: Let f be a Pythagorean neutrosophic pre-homeomorphism. Then, f and f^{-1} are Pythagorean neutrosophic pre-continuous functions. Since every Pythagorean neutrosophic closed set of $(X, \mathfrak{T})$ is a Pythagorean neutrosophic pre-closed set of $(X, \mathfrak{T})$, this implies that f is a Pythagorean neutrosophic pre-closed function. □

The following example shows that the composition of two Pythagorean neutrosophic pre-homeomorphisms need not be a Pythagorean neutrosophic pre-homeomorphism.

Example 3.13. Let $X = \{q, w\}$, $Y = \{e, r\}$ and $Z = \{t, y\}$. Then, $\mathfrak{T} = \{0_N, \mathfrak{U}, 1_N\}$, $\mathfrak{Y} = \{0_N, \mathfrak{V}, 1_N\}$ and $\mathfrak{Z} = \{0_N, \mathfrak{W}, 1_N\}$ are Pythagorean neutrosophic topological spaces on X, Y and Z respectively, where $\mathfrak{U} = \langle x, (0.1, 0.3, 0.5), (0.3, 0.5, 0.7)\rangle$, $\mathfrak{V} = \langle y, (0.2, 0.7, 0.9), (0.3, 0.6, 0.7)\rangle$ and $\mathfrak{W} = \langle z, (0.7, 0.5, 0.2), (0.7, 0.7, 0.2)\rangle$. We define the function $f : (X, \mathfrak{T}) \to (Y, \mathfrak{Y})$ as $f(q) = e$ and $f(w) = r$. Besides, we define the function $g : (Y, \mathfrak{Y}) \to (, \mathfrak{Z})$ as $g(e) = t$ and $g(r) = y$. We can see that f and g are Pythagorean neutrosophic pre-homeomorphism, but $g \circ f$ is not a Pythagorean neutrosophic pre-homeomorphism.

Definition 3.14. Let $f : (X, \mathfrak{T}) \to (Y, \mathfrak{Y})$ be a function where $(X, \mathfrak{T})$ and $(Y, \mathfrak{Y})$ are Pythagorean neutrosophic topological spaces. Then, f is said to be Pythagorean neutrosophic pre-irresolute if $f^{-1}(V)$ is a Pythagorean neutrosophic pre-open set in X for every Pythagorean neutrosophic pre-open set V in Y.

Definition 3.15. Let $f : (X, \mathfrak{T}) \to (Y, \mathfrak{Y})$ be a bijection function where $(X, \mathfrak{T})$ and $(Y, \mathfrak{Y})$ are Pythagorean neutrosophic topological spaces. Then, f is said to be Pythagorean neutrosophic prei-homeomorphism if f and f^{-1} are Pythagorean neutrosophic pre-irresolute functions.

Theorem 3.16. *Every Pythagorean neutrosophic prei-homeomorphism is a Pythagorean neutrosophic pre-homeomorphism.*

Proof. Let $f : (X, \mathfrak{T}) \to (Y, \mathfrak{Y})$ be a bijection and Pythagorean neutrosophic prei-homeomorphism function. Suppose that B is a Pythagorean neutrosophic closed set of $(Y, \mathfrak{Y})$, this implies that B is a Pythagorean neutrosophic pre-closed set of $(Y, \mathfrak{Y})$. Now, since f is Pythagorean neutrosophic irresolute, $f^{-1}(B)$ is a Pythagorean neutrosophic pre-closed set of $(X, \mathfrak{T})$. Indeed, f is a Pythagorean neutrosophic pre-continuous function. therefore, f and f^{-1} are Pythagorean neutrosophi pre-continuous functions and then f is Pythagorean neutrosophic pre-homeomorphism. □

The following example shows that the converse of the above Theorem need not be true.

Example 3.17. Let $X = \{q, w\}$ and $Y = \{e, r\}$. Then, $\mathfrak{T} = \{0_N, \mathfrak{U}_1, \mathfrak{U}_2, 1_N\}$ and $\mathfrak{Y} = \{0_N, \mathfrak{V}, 1_N\}$ are Pythagorean neutrosophic topological spaces on X and Y respectively, where $\mathfrak{U}_1 = \langle x, (0.2, 0.4, 0.6), (0.3, 0.3, 0.3)\rangle$, $\mathfrak{U}_2 = \langle x, (0.4, 0.7, 0.9), (0.1, 0.1, 0.3)\rangle$ and $\mathfrak{V} = \langle y, (0.4, 0.7, 0.9), (0.1, 0.2, 0.3)\rangle$. Then, we define the function $f : (X, \mathfrak{T}) \to (Y, \mathfrak{Y})$ as $f(q) = e$ and $f(w) = w$. We can see that f is a Pythagorean neutrosophic pre-homeomorphism, but it is not a Pythagorean neutrosophic prei-homeomorphism.

Theorem 3.18. *If $f : (X, \mathfrak{T}) \to (Y, \mathfrak{Y})$ and $g : (Y, \mathfrak{Y}) \to (Z, \mathfrak{Z})$ are Pythagorean neutrosophic prei-homeomorphisms, then $g \circ f : (X, \mathfrak{T}) \to (Z, \mathfrak{Z})$ is a Pythagorean neutrosophic prei-homeomorphism.*

Proof. Let f and g be two Pythagorean neutrosophic pre-homeomorphisms. Now, suppose that B is a Pythagorean neutrosophic pre-closed set of $(Z, \mathfrak{Z})$, then $g^{-1}(B)$ is a Pythagorean neutrosophic pre-closed set of $(Y, \mathfrak{Y})$. Then by hypothesis, $f^{-1}(g^{-1}(B))$ is a Pythagorean neutrosophic pre-closed set of $(X, \mathfrak{T})$. Therefore, $g \circ f$ is a Pythagorean neutrosophic pre-irresolute function Now, let β be a Pythagorean neutrosophic pre-closed set of $(X, \mathfrak{T})$. By assumption, $f(\beta)$ is a Pythagorean neutrosophic pre-closed set of $(Y, \mathfrak{Y})$. Then, by hypothesis, $g(f(\beta))$ is a Pythagorean neutrosophic pre-closed set of $(Z, \mathfrak{Z})$. This implies that $g \circ f$ is a Pythagorean neutrosophic pre-irresolute function and then $g \circ f$ is a Pythagorean neutrosophic prei-homeomorphism. □

4 Conclusion

In this paper, we carried out a new classes of open sets in Pythagorean neutrosophic topological spaces, these sets were called Pythagorean neutrosophic pre-open sets, we found out some of their properties, as well as, we proved that every Pythagorean neutrosophic pre-open set is a Pythagorean neutrosophic b-open set. On the other hand, new variants of continuity were defined and studied which are namely Pythagorean neutrosophic pre-continuous functions, Pythagorean neutrosophic pre-irresolute functions, Pythagorean neutrosophic pre-homeomorphisms, Pythagorean neutrosophic prei-homeomorphisms.

Acknowledgement

We want to thank referees for their useful suggestions which helped us to improve our manuscript.

References

[1] I. Arockiarani, R. Dhavaseelan, S. Jafari, M. Parimala. On some notations and functions in neutrosophic topological spaces, Neutrosophic sets and Systems, 16, 16–19, 2017.

[2] K. Atannasov. Intuitionistic fuzzy sets, Fuzzy sets and Systems, 20(1), 87–96, 1965.

[3] K. Atannasov. Intuitionistic fuzzy sets, Springer Physica-Verlag, Heidelberg, 1999.

[4] V. Banu, S. Chandrasekar. Neutrosophic αgs Continuity And Neutrosophic gs Irresolute Maps, Neutrosophic sets and Systems, 27, 163–170, 2019.

[5] C. Chang. Fuzzy topological spaces, J. Math. Anal Appl., 24(1), 182–190, 1968.

[6] R. Jansi, K. Mohana, F. Smarandache. Correlation Measure for Pythagorean Neutrosophic Sets with T and F as Dependent Neutrosophic Components, Neutrosophic sets and Systems, 30, 202–212, 2019.

[7] T. Sneha, F. Nirmala .Pythagorean neutrosophic b-open and semi-open sets in Pythagorean neutrosophic topological spaces, Infokara Research, 9(1), 860–872, 2020.

[8] A. Salama, S. Albowi. Neutrosophic set and Nutrosophic topological spaces, IOSR Jouranl of Mathematics, 3(4), 31–35, 2012.

[9] F. Smarandache. A unifying field in logics-neotrusophic: Neutrosophic probability, set and logic, Rehoboth: American Research Press, 1999.

[10] Z. Xu, R. Yagar. Some geometric aggregation operations based on intuitionistic fuzzy sets , Int. J. Gen. Syst., 35, 417–433, 2006.

[11] R. Yager, A. Abbasov. Pythagorean membership grades, complex numbers and decision making, Int. J. Intell. Syst., 28, 436–452, 2013.

[12] L. Zadeh. Fuzzy sets, Inform and control, 8, 1965.