Some new results on the decomposition of nano continuity

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Abstract

The main purpose of this paper is to introduce the concepts of Nη-sets, N*η-sets, N$\eta\zeta$-sets, Nη-continuity and N$\eta\zeta$-continuity and to obtain a decomposition of nano continuity in nano topological spaces..

2020 Mathematics Subject Classification: 54C10, 54A05, Secondary 54D15, 54D30.

Keywords: NαB-sets, Nη-sets, N$\eta\zeta$-set, Nη-continuity and N$\eta\zeta$-continuity.

1 Introduction

Jayalakshmi and Janaki [5] introduced and studied the notions of Nt-sets, NA-sets and NB-sets in nano topological spaces. Various decompositions of nano continuous functions are given in [[2], [3]]. In this paper, we introduce the notions of NαB-sets, Nη-sets, N$\eta\zeta$-sets, NαB-continuity, Nη-continuity and N$\eta\zeta$-continuity and obtain decomposition of Nano continuity.

2 Preliminaries

2.1 Definition

A subset M of a space (U, $\tau_R(X)$) is called:

1. Nano α-open set [6] if M $\subseteq$ Nint(Ncl(Nint(M))).
2. Nano semi-open set [6] if M $\subseteq$ Ncl(Nint(M)).
3. Nano pre-open set [6] if M $\subseteq$ Nint(Ncl(M)).
4. Nano β-open set [12] if M $\subseteq$ Ncl(Nint(Ncl(M))).
5. Nano regular-open set [6] if M $=$ Nint(Ncl(M)).

The complements of the above mentioned Nano open sets are called their respective Nano closed sets.
The Nano α-closure [4] of a subset M of U denoted by Nαcl(M) is defined to be the intersection of all Nano α-closed sets of (U, $\tau_R(X)$) containing M.

2.2 Definition

A subset M of a space (U, $\tau_R(X)$) is called:

1. a Nt-set [5] if Nint(Ncl(A)) = Nint(A).
2. an NA-set [5] if M = S ∩ G where S is Nano open and G is a Nano regular closed set.
3. a NB-set [5] if M = S ∩ G where S is Nano open and G is a Nt-set.
4. a Nano locally closed set [1] if M = S ∩ G where S is Nano open and G is Nano closed.
5. Nano nowhere dense (briely, N-nowhere dense) [8] if Nint(Ncl(M)) = ϕ.

The collection of Nt-sets (resp. NA-sets, NB-sets, Nano locally closed sets) in U is denoted by Nt(U) (resp. NA(U), NB(U), NLC(U)).

2.3 Definition

A subset M of a space (U, $\tau_R(X)$) is called an Nano α-generalized closed (briefly, Nαg-closed) set [16] if Nαcl(M) ⊆ T whenever M ⊆ T and T is Nano open in (U, $\tau_R(X)$). The collection of all Nαg-closed sets is denoted by Nαgc(U).

2.4 Theorem

1. Every Nano closed is Nt-set but not conversely [5].
2. Every Nano α-closed set is Nano semi-closed but not conversely [10].
3. Every Nt-set is NB-set but not conversely [5].
4. Every NA-set is NB-set but not conversely [5].

2.5 Theorem

[5], In a space U, the followings hold:

1. M is Nt-set if and only if it is Nano semi closed.
2. If M and N are two Nt-sets, then A ∩B is a Nt-set.

2.6 Definition

[11, 15] A subset M of a space (U, $\tau_R(X)$) is called nano semi-regular if M is Nano semi-open and a Nt-set.

2.7 Theorem

[15]

1. Every NA-continuous function is NAB-continuous.
2. Every NAB-continuous function is NB-continuous.

2.8 Definition

[15] A subset M of a space (U, $\tau_R(X)$) is called an Nano NDB-set if M has Nano nowhere dense boundary.

2.9 Definition

A map f : (U, $\tau_R(X)$) $\rightarrow$ (L, $\tau'_R(Y)$) is called:

1. Nano continuous [7] if f^{-1}(V) is a Nano open set of U for every Nano open set V of L.
2. Nano semi-continuous [9, 13] if f^{-1}(V) is a Nano semi-open set of U for every Nano open set V of L.
3. Nano pre-continuous [9, 13] if f^{-1}(V) is a Nano pre-open set of U for every Nano open set V of L.
4. Nano α-continuous [9] if f^{-1}(V) is an Nano α-open set in U for every Nano open set V of L.
5. Nano β-continuous [10] if f^{-1}(V) is an Nano β-open set in U for every Nano open set V of L.
6. NLC-continuous [1] if f^{-1}(V) is an Nano locally closed set in U for every Nano open set V of L.
7. Nano α-irresolute [17] if f^{-1}(V) is an Nano α-open set in U for every Nano α-open set V of L.
8. A-continuous [14, 15] if f^{-1}(V) is an NA-set in U for every Nano open set V of L.
9. B-continuous [14, 15] if f^{-1}(V) is an NB-set in U for every Nano open set V of L.

3 Nη-sets and N$\eta\zeta$-sets

In this section, Introduce the study of notions of Nη-sets and Nζ-sets in nano topological spaces.

3.1 Definition

A subset M of a space U is called:

1. an NαB-set if M = S $\cap$ G where S is Nano α-open and G is a Nt-set.
2. an Nη-set if M = S $\cap$ G where S is Nano open and G is a Nano α-closed set.
3. an N$\eta\zeta$-set if M = S $\cap$ G where S is Nano open and G is an Nano clopen set.

The collection of all NαB-sets (resp. Nη-sets, N$\eta\zeta$-sets) in U will be denoted by NαB(U) (resp. Nη(U), N$\eta\zeta$(U)).

3.2 Theorem

N$\eta\zeta$-set is NA-set but not conversely.

Proof: Let A be N$\eta\zeta$-set. Then A = S ∩ G, where S is Nano open and G is Nano clopen set. Since every Nano clopen set is Nano regular open, G is NA-set. Hence A is NA-set.

3.3 Example

Let U = {1, 2, 3, 4} with U/ R= {{1}, {4}, {2, 3}} and X= {1, 3}. The Nano topology $\tau_R(X)$= {ϕ, {1}, {2, 3}, {1, 2, 3}, U}. Then NA-sets are ϕ, U, {1}, {1, 4}, {2, 3}, {1, 2, 3}, {2, 3, 4} and N$\eta\zeta$-sets are ϕ, U, {1}, {2, 4}, {1, 2, 3}. It is clear that {1, 4} is N$\eta\zeta$-set but it is not NA-set.

3.4 Theorem

Every NA-set is Nano locally closed set but not conversely.

Proof: Let A be NA-set. Then A = S ∩ G, where S is Nano open and G is Nano regular closed set. Since every Nano regular closed set is Nano closed set, G is Nano closed. Hence A is Nano locally closed set.

3.5 Example

Let U and $\tau_R(X)$ as in the Example 3.3. Then Nano locally closed sets are ϕ, U, {1}, {4}, {1, 4}, {2, 3}, {1, 2, 3}, {2, 3, 4}. It is clear that {4} is Nano locally closed set but it is not NA-set.

3.6 Theorem

Every NB-set is NαB-set but not conversely.

Proof: Let A be NB-set. Then A = S ∩ G, where S is Nano open and G is Nt-set. Since every Nano open set is Nano α-open set, S is Nano α-open set. Hence A is NαB-set.

3.7 Example

Let U = {1, 2, 3, 4} with U/ R= {{3}, {4}, {1, 2}} and X= {2}. The Nano topology $\tau_R(X)$= {ϕ, {1, 2}, U}. Then NB-sets are ϕ, U, {3}, {4}, {1, 2}, {3, 4} and NαB-sets are ϕ, U, {3}, {4}, {1, 2}, {3, 4}, {1, 2, 3}, {1, 2, 4}. It is clear that {1, 2, 3} is NαB-set but it is not NB-set.

3.8 Theorem

Every Nano locally closed set is Nη-set but not conversely.

Proof: Let A be Nano locally closed set. Then A = S ∩ G, where S is Nano open and G is Nano closed set. Since every Nano closed set is Nano α-closed set, G is Nano α-closed. Hence A is Nη-set.

3.9 Example

Let U and $\tau_R(X)$ as in the Example 3.7. Then Nano locally closed sets are ϕ, U, {1, 2}, {3, 4} and Nη-sets are ϕ, U, {3}, {4}, {1, 2}, {3, 4}. It is clear that {3} is Nη-set but it is not Nano locally closed set.

3.10 Theorem

Every Nη-set is NαB-set but not conversely.

Proof: Let A be Nη-set. Then A = S $\cap$ G, where S is Nano open and G is Nano α-closed set. Since every Nano open set is Nano α-open set, S is Nano α-open and every Nano α-closed set is Nt-set by Theorems 2.4(2), 2.5(1), G is Nt-set. Hence A is NαB-set.

3.11 Example

Let U and $\tau_R(X)$ as in the Examples 3.7 and 3.9. It is clear that {1, 2, 4} is NαB-set but it is not Nη-set.

3.12 Remark

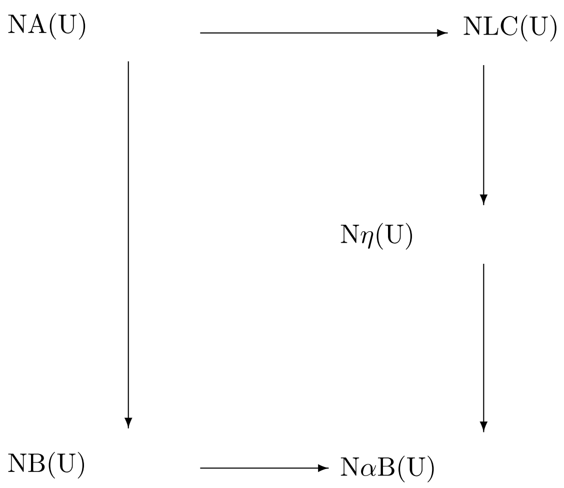
The notions of Nη-sets and Nαg-closed sets are independent.

3.13 Example

Let U and $\tau_R(X)$ as in the Example 3.9. Then Nαg-closed sets ϕ, U, {3}, {4}, {1, 3}, {1, 4}, {2, 3}, {2, 4}, {3, 4}, {1, 2, 3}, {1, 2, 4}, {1, 3, 4}, {2, 3, 4}. The set {1, 4} is Nαg-closed set but not an Nη-set and also the set {1, 2} is an Nη-set but not a Nαg-closed set in (U, $\tau_R(X)$).

3.14 Remark

From the above discussions, we obtain the following diagram where A$\rightarrow$B represents A implies B, but not conversely.



3.15 Theorem

For a subset M of a space U, the following are equivalent.

1. M is an Nη-set.
2. M $=$ S $\cap$ Nαcl(M) for some Nano open set S.

Proof: (1)$\Rightarrow$ (2) Since M is an Nη-set, then M $=$ S $\cap$ G, where S is Nano open and G is Nano α-closed. So, M $\subset$ S and M $\subset$ G. Hence Nαcl(M) $\subset$ Nαcl(G). Therefore M $\subset$ S $\cap$ Nαcl(M) $\subset$ S $\cap$ Nαcl(G) $=$ S $\cap$ G $=$ M. Thus, M $=$ S $\cap$ Nαcl(M).
(2)$\Rightarrow$ (1) It is obvious because Nαcl(M) is Nano α-closed. (Since A is Nano α-closed if and only if A $=$ Nαcl(A)).

3.16 Theorem

For a subset M of a space U, the following are equivalent:

1. M is Nano α-closed.

2. M is an Nη-set and Nαg-closed.

Proof: (1) $\Rightarrow$ (2). This is obvious.
(2) $\Rightarrow$ (1). Since M is an Nη-set, then by Theorem 3.15, M = S $\cap$ Nαcl(M) where S is an Nano open in U. So, M $\subset$ S and since M is Nαg-closed, then Nαcl(M) $\subset$ S. Therefore, Nαcl(M) $\subset$ S $\cap$ Nαcl(M) = M. Hence, M is Nano α-closed.

3.17 Theorem

For a subset M of a Nano topological space (U, $\tau_R(X)$), the following are equivalent:

1. M is Nano open.
2. M is an N$\eta\zeta$-set.
3. M is Nano α-open and an NA-set.
4. M is Nano pre open and an NA-set.
5. M is Nano α-open and an Nη-set.
6. M is Nano α-open and an Nano locally closed.
7. M is an Nano pre-open and Nano locally closed.
8. M is Nano pre-open and an Nη-set.
9. M is Nano pre-open and an NαB-set.

Proof: (1) $\Rightarrow$ (2). Since M is Nano open and M = M $\cap$ U, where U is Nano clopen then M is an N$\eta\zeta$-set.
(2) $\Rightarrow$ (3) is trivial.
(3) $\Rightarrow$ (4) is trivial [Every Nano α open set is Nano pre open set but not conversely [10]].
(4) $\Rightarrow$ (5). Since M is an NA-set, then M is Nano semi-open. Now, M is both Nano semi-open and Nano pre-open and hence Nano α-open. The second part is trival.
(5) $\Rightarrow$ (6). Since M is an Nη-set, then M = S $\cap$ Nαcl(M), where S is Nano open. Therefore, since M is Nano β-open, Ncl(Nαcl(M)) = Ncl(M $\cup$(Ncl(Nint(Ncl(M)))) = Ncl(Ncl(Nint(Ncl(M)))) = Ncl(Nint(Ncl(M))) = Nαcl(M). Hence Nαcl(M) is Nano closed and M is Nano locally closed.
(6) $\Rightarrow$ (7), (7) $\Rightarrow$ (8) and (8) $\Rightarrow$ (9) are trivial.
(9)$\Rightarrow$ (1) Let M be Nano open. Every Nano open is Nano pre-open and M = M $\cap$ U. Thus M is NαB-set. The second part. Let M be NαB-set. Then we have M = S $\cap$ G, where S is Nano α open and G is Nano Nt-set. Since M is Nano pre-open, M $\subset$ Nint(Ncl(M)). Hence M = S $\cap$ G = (S $\cap$ G) $\cap$ S $\subset$ [Nint(Ncl(S) $\cap$ Nint(F)] $\cap$ S = S $\cap$ Nint(G). Therefore M is Nano open.

3.18 Theorem

For a subset M of a nano topological space (U, $\tau_R(X)$), the following are equivalent:

1. M is an NA-set.

2. M is Nano semi-open and an Nη-set.
3. M is Nano β-open and Nano locally closed.
4. M is Nano β-open and an Nη-set.

Proof: (1) $\Rightarrow$ (2). This is trivial.
(2) $\Rightarrow$ (3). Since M is an Nη-set, then M = S $\cap$ Nαcl(M) where S is Nano open. Also, since M is Nano semi-open and so Nano β-open, then Ncl(Nαcl(M)) = Ncl(M $\cup$ (Ncl(Nint(Ncl(M))))) =Ncl(Ncl(Nint(Ncl(M)))) = Ncl(Nint(Ncl(M))) = Nαcl(M). Hence, Nαcl(M) is Nano closed and M is Nano locally closed.
(3) $\Rightarrow$ (4). This is trivial.
(4) $\Rightarrow$ (1). Since M is an Nη-set, then M = S $\cap$ Nαcl(M), where S is Nano open. Also, since M is Nano β-open, then Nαcl(M) = Ncl(Nint(Ncl(M))). So, Ncl(Nint(Nαcl(M))) = Ncl(Nint(Ncl(Nint(Ncl(M))))) = Ncl(Nint(Ncl(M))) = Nαcl(M). Hence, Nαcl(M) is Nano regular closed and M is an NA-set.

3.19 Theorem

For a subset M of a nano topological space (U, $\tau_R(X)$), the following are equivalent:

1. M is Nano α-closed set.
2. M is Nano pre-closed and an Nη-set.
3. M is Nano pre-closed and an NαB-set.

Proof: (1) $\Rightarrow$ (2). Every Nano α-closed set is Nano pre-closed. Since M = M $\cap$ U, where M is Nano α-closed and U is Nano open, then M is an Nη-set.
(2) $\Rightarrow$ (3). Every Nη is a NαB-set.
(3) $\Rightarrow$ (1). First part. Let M be Nano α-closed. Every Nano α closed is Nano pre-closed and M = M $\cap$ U. Thus M is NαB-set. Second part. Let M be NαB-set. Then we have M = S $\cap$ G, where S is Nano α closed and G is Nano Nt-set. Since M is Nano pre-closed, Ncl(Nint(M)) $\subset$ M. Hence M = S $\cap$ G = (S $\cap$ G) $\cap$ S $\subset$ [Ncl(Nint(S) $\cap$ G] $\cap$ S = Nαclosed $\cap$ S = Nano α-closed. Therefore M is Nano α-closed. [Nano pre-closed and Nano semi-closed(= Nt-set) is equivalent Nano α-closed [10]].

3.20 Remark

In a space U, the intersection of two Nη-sets is an Nη-set.

3.21 Remark

Union of two Nη-sets need not be an Nη-set as seen from the following example.

3.22 Example

Let U and $\tau_R(X)$ as in the Example 3.9. The sets {1, 2}, {3} are Nη-sets in (U, $\tau_R(X)$) but their union {1, 2, 3} is not an Nη-set in (U, $\tau_R(X)$).

3.23 Theorem

Let M be a subset of a Nano topological space (U, $\tau_R(X)$). If M $\in$ Nη(U), then Nαcl(M) $-$ M is Nano α-closed, M $\cup$ (U $-$ Nαcl(M)) is Nano α-open and M $\subseteq$ Nano αint(M $\cup$ (U $-$ Nαcl(M))).

Proof: First, if M $\in$ Nη(U), then from Theorem 3.15 we have that M = S $\cap$ Nαcl(M) for some Nano open set S. Therefore Nαcl(M) $-$ M = Nαcl(M)$-$ (S $\cap$ Nαcl(M)) = Nαcl(M) $\cap$ (U $-$ (S $\cap$ Nαcl(M))) = Nαcl(M) $\cap$ ((U$-$ S)$\cup$ (U$-$Nαcl(M))) = (Nαcl(M) $\cap$(U$-$ S)) $\cup$ (Nαcl(M) $\cap$ (U$-$ Nαcl(M))) = (Nαcl(M) $\cap$(U$-$ S)) $\cup$ ϕ = Nαcl(M) $\cap$ (U$-$ S) which is Nano α-closed.
Second, since Nαcl(M) $-$ M is Nano α-closed, then U $-$ (Nαcl(M) $-$ M) is Nano α-open. Therefore, U $-$ (Nαcl(M) $-$ M) = U $-$ (Nαcl(M) $\cap$ (U $-$ M)) = M $\cup$(U $-$ Nαcl(M)). Finally, since M $\cup$ (U $-$ Nαcl(M)) is Nano α-open, then M $\subseteq$ M $\cup$ (U $-$ Nαcl(M)) = Nano αint(M $\cup$ (U $-$ Nαcl(M))).

3.24 Example

Let U and $\tau_R(X)$ as in the Example 3.7. The set M = {1, 2, 3}. It is observed that Nαcl(M)) $-$ M = {4} is Nano α-closed and M $\cup$ (U $-$ Nαcl(M)) = M is Nano α-open but M $\notin$ Nη(U).

3.25 Theorem

For a space U, the following are equivalent:

1. U is Nano indiscrete.
2. The Nη-sets in U are only the trivial ones.
3. The N$\eta\zeta$-sets in U are only the trivial ones.

Proof: (1) $\Rightarrow$ (2). If M is an Nη-set, then M = S $\cap$ G, where S is Nano open and G is Nano α-closed. If M $\neq \phi$, then S $\neq \phi$ and by (1), S = U. Thus M = G and so M $\supset$ Ncl(Nint(Ncl(M))) = Ncl(Nint(U)) = U. Hence M = U.
(2) $\Rightarrow$ (3). Every N$\eta\zeta$-set is an Nη-set.
(3) $\Rightarrow$ (1). Since every Nano open set is an N$\eta\zeta$-set, then by (3) the Nano open sets in U are only the trivial ones, i.e., U is Nano indiscrete.

3.26 Theorem

For a space U, the following are equivalent:

1. U is Nano discrete.

2. Every subset of U is an N$\eta\zeta$-set.

Proof: (1) $\Rightarrow$ (2). By (1) any subset M of U is Nano clopen. Hence, M is an N$\eta\zeta$-set.
(2) $\Rightarrow$ (1). By (2) every singleton {u} of U is an N$\eta\zeta$-set and hence Nano open. Thus U is Nano discrete.

3.27 Theorem

For a subset M of a nano topological space (U, $\tau_R(X)$), the following are equivalent:

1. M is Nano α-closed.
2. M is Nano pre-closed and an Nη-set.
3. M is Nano pre-closed and an NB-set.
4. M is Nano pre-closed and an Nano NDB-set.

Proof: (1) $\Rightarrow$ (2). Every Nano α-closed set is Nano pre-closed. Since M = M $\cap$ U, where M is Nano α-closed and U is Nano open, then M is an Nη-set.
(2) $\Rightarrow$ (3). Every Nη-set is a NB-set.
(3) $\Rightarrow$ (4). Every NB-set is an Nano NDB-set [15].
(4) $\Rightarrow$ (1). Let M be an Nano NDB-set and P = U $-$ M. Then Ncl(Nint(NFr(P))) = ϕ and Nint(NFr(P)) = ϕ. Therefore, Nint(NFr(P)) = Nint(Ncl(P) $\cap$ Ncl(U $-$ P)) = NintNcl(P) $\cap$ NintNcl(U $-$ P) = NintNcl(P) $\cap$ (U $-$ Ncl(Nint(P))) = ϕ. So, NintNcl(P) $\subset$ Ncl(Nint(P)). Since P is Nano pre-open, then P $\subset$ Nint(Ncl(P)) $\subset$ Ncl(Nint(P)). Therefore, P is Nano semi-open and hence M is Nano α-closed.

4 Nη-continuity and N$\eta\zeta$-continuity

4.1 Definition

A map f : (U, $\tau_R(X)$) $\rightarrow$ (L, $\tau'_R(Y)$) is called:

1. NαB-continuous if f^{-1}(V) is an NαB-set in U for every Nano open set V of L.
2. Nη-continuous if f^{-1}(V) is an Nη-set in U for every Nano open set V of L.
3. N$\eta\zeta$-continuous if f^{-1}(V) is an N$\eta\zeta$-set in U for every Nano open set V of L.

4.2 Theorem

Every N$\eta\zeta$-continuous is NA-continuous but not conversely.

Proof: It is follows from Theorem 3.2.

4.3 Example

Let U = {1, 2, 3}, with U/ R= {{3}, {1, 2}, {2, 1}} and X= {1, 2}. Then the Nano topology $\tau_R(X)$ = {ϕ, {1, 2}, U}. Let L = {1, 2, 3} with L/ R'= {{3}, {1, 2}} and Y= {3}. Then Nano topology $\tau'_R(Y)$ = {ϕ, {3}, L}. Then N$\eta\zeta$-sets are ϕ, U and NA-sets are ϕ, U, {3}, {1, 2}. Define f : (U, $\tau_R(X)$) $\rightarrow$ (L, $\tau'_R(Y)$) be the identity map. Then NA-continuous but not N$\eta\zeta$-continuous, since f^{-1}({3})= {3} is not N$\eta\zeta$-set.

4.4 Theorem

Every NA-continuous is NLC-continuous but not conversely.

Proof: It is follows from Theorem 3.4.

4.5 Example

Let U = {1, 2, 3}, with U/ R= {{1}, {2, 3}} and X= {1}. Then the Nano topology $\tau_R(X)$ = {ϕ, {1}, U}. Let L = {1, 2, 3} with L/ R'= {{1}, {2, 3}, {3, 2}} and Y= {2, 3}. Then Nano topology $\tau'_R(Y)$ = {ϕ, {2, 3}, L}. Then NA-sets are ϕ, U, {1} and NLC-sets are ϕ, U, {1}, {2, 3}. Then NLC-continuous but not NA-continuous, since f^{-1}({2, 3})= {2, 3} is not NA-set.

4.6 Theorem

Every NB-continuous is NαB-continuous but not conversely.

Proof: It is follows from Theorem 3.6.

4.7 Example

Let U, $\tau_R(X)$, and f as in the Example 4.5. Let L = {1, 2, 3} with L/ R'= {{2}, {1, 3}} and Y= {1, 2}. Then Nano topology $\tau'_R(Y)$ = {ϕ, {2}, {1, 3}, L}. Then NαB-sets are ϕ, U, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}. Then NαB-continuous but not NB-continuous, since f^{-1}({1, 3})= {1, 3} is not NB-set.

4.8 Theorem

Every NLC-continuous is Nη-continuous but not conversely.

Proof: It is follows from Theorem 3.8.

4.9 Example

Let U, $\tau_R(X)$, and f as in the Example 4.5. Let L = {1, 2, 3} with L/ R'= {{2}, {1, 3}} and Y= {2}. Then Nano topology $\tau'_R(Y)$ = {ϕ, {2}, L}. Then Nη-sets are ϕ, U, {1}, {2}, {3}, {2, 3}. Then Nη-continuous but not NLC-continuous, since f^{-1}({2})= {2} is not NLC-set.

4.10 Theorem

Every Nη-continuous is NαB-continuous but not conversely.

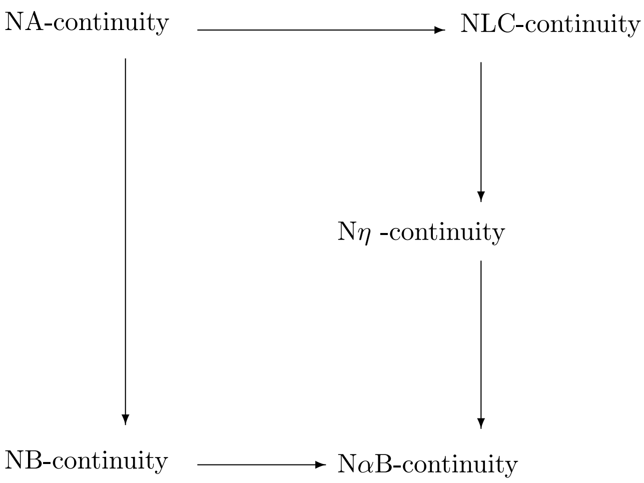
Proof: It is follows from Theorem 3.10.

4.11 Example

Let U, $\tau_R(X)$, and f as in the Example 4.5. Let L = {1, 2, 3} with L/ R'= {{3}, {1, 2}} and Y= {2, 3}. Then Nano topology $\tau'_R(Y) = \{\phi$, {3}, {1, 2}, L}. Then NαB-continuous but not Nη-continuous, since f^{-1}({1, 2})= {1, 2} is not Nη-set.

4.12 Remark

From the above discussions we obtain the following diagram where A→B represents A implies B, but not conversely.



4.13 Theorem

Let f : (U, $\tau_R(X)$) → (L, $\tau'_R(Y)$) be a Nano continuous and Nano α-irresolute map. If V is an Nη-set in (L, $\tau'_R(Y)$), then f^{-1}(V) is an Nη-set in (U, $\tau_R(X)$).

Proof: Let V be an is an Nη-set in (L, $\tau'_R(Y)$). Then V= S ∩ G, where S is Nano open in (L, $\tau'_R(Y)$) and G is Nano α-closed in (L, $\tau'_R(Y)$). Therefore, f^{-1}(V) = f^{-1}(S ∩ G) = f^{-1}(S) ∩ f^{-1}(G). Since f is Nano continuous then f^{-1}(S) is Nano open in (U, $\tau_R(X)$). Also since f is Nano α-irresolute, then f^{-1}(G) is Nano α-closed in (U, $\tau_R(X)$). Hence , f^{-1}(V) is an Nη-set in (U, $\tau_R(X)$).

4.14 Theorem

For a map f : (U, $\tau_R(X)$) → (L, $\tau'_R(Y)$), the following are equivalent.

1. f is Nano α-continuous.
2. f is Nη-continuous and Nαg-continuous.

Proof: From Theorem 3.16, the proof is immediate.

4.15 Theorem

For a map f : (U, $\tau_R(X)$) → (L, $\tau'_R(Y)$), the following are equivalent.

1. M is Nano continuous.
2. M is an N$\eta\zeta$-continuous.
3. M is Nano α-continuous and an NA-continuous.
4. M is Nano pre continuous and an NA-continuous.
5. M is Nano α-continuous and an Nη-continuous.
6. M is Nano α-continuous and an NLC-continuous.
7. M is an pre-continuous and NLC-continuous.
8. M is Nano pre-continuous and an Nη-continuous.
9. M is Nano pre-continuous and an NαB-continuous.

Proof: From Theorem 3.17, the proof is immediate.

4.16 Theorem

For a map f : (U, $\tau_R(X)$) → (L, $\tau'_R(Y)$), the following are equivalent.

1. M is an NA-continuous.
2. M is Nano semi continuous and an Nη-continuous.
3. A is Nano β-continuous and Nano locally continuous.
4. A is Nano β-continuous and an Nη-continuous.

Proof: From Theorem 3.18, the proof is immediate.

4.17 Theorem

For a map f : (U, $\tau_R(X)$) $\rightarrow$ (L, $\tau'_R(Y)$), the following are equivalent.

1. M is Nano α-continuous.
2. M is Nano pre-continuous and an Nη-continuous.
3. M is Nano pre-continuous and an NαB-continuous.

Proof: From Theorem 3.19, the proof is immediate.

Acknowledgement

I thank to referees for giving their useful suggestions and help to improve this manuscript.

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